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Order Amid Sparse Data: Patterns of Lake Level Changes in North America During the Late Quaternary[1]

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Water-level fluctuations in closed-basin lakes can be used to reconstruct past hydrological changes, and the recognition of spatially coherent patterns in lake behavior provides evidence for changes in climate. The geological records of water level in many lakes, particularly those in arid regions, are by nature incomplete. The fragmentary nature of the data poses special problems for comparison of records and identification of regions where lakes behave similarly. An unconventional method of assessing similarity in the behavior of lakes is used with multidimensional scaling to place lakes in a low-dimensional space. Weights are used to reflect the amount of information available for each particular comparison. The similarity measure is based on evidence for changes in lake depth between successive time intervals and on independent evidence for the direction of change at any given time. Groups (clusters) of lakes in the low-dimensional space are identified by mutual proximity. The method was applied to a set of 65 Late Quaternary lake-level records from North America. About one-third of the lakes had too little weight to be placeable, about one-third were in clusters, and about one-third showed unique behavior. Those lakes which clustered showed four distinct types of record, characteristic of well-defined geographic regions. This ability to distinguish spatially coherent patterns on internal evidence alone strengthens the basis for using lake-level records for regional palaeoclimatic reconstructions.

KEY WORDS: climatic change, missing data, dissimilarity coefficient, multidimensional scaling, cluster recognition.

INTRODUCTION

Water-level fluctuations in closed-basin lakes provide evidence of climatic changes on a Quaternary time scale (Smith and Street-Perrott, 1983; Street-Perrott and Harrison, 1985). Spatially coherent patterns in lake behavior through

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time, identified by inspection of maps, have been used to indicate displacements of major features of the general atmospheric circulation as reflected by changes in air mass trajectories (Harrison and Metcalfe, 1985a,b). The more objective identification of spatial patterns in lake-level data is not easy, because the record of past lake levels is semi-quantitative and usually incomplete, and because lakes are unevenly distributed geographically. The standard methods for handling missing data in multivariate analysis are not designed to deal with intrinsically sparse matrices. Furthermore, the form of the data rules out standard analytical techniques. The general problem is one identified by Skellam (1972): Data analysis techniques are devised with particular assumptions that are not true of all types of data. The more awkward or unusual the data set, the more important it is to devise an ''indigenous'' method of analysis rather than to use a standard (''exotic'') one.

In this article, an indigenous method is described. First, a measure of the dissimilarity in the temporal behavior of two lakes, and the weight to be given to the value of the measure in any given pairwise comparison, is derived. The dissimilarity measure and its weight explicitly allow for incompleteness of the record. For some comparisons, the information is so sparse that no dissimilarity measure can be calculated. Therefore, an incomplete triangular matrix of comparisons exists. Multidimensional scaling (Kruskal, 1964a,b), which can deal with weighted and incomplete dissimilarity matrices and requires few assumptions about their properties, is used to produce a set of coordinates for the lakes in two- to four-dimensional space. Lakes that are close together in this ordination space are those that have behaved similarly during the period of comparison. Clusters corresponding to distinct temporal patterns are recognized on the basis of distances between lakes in the ordination space.

This method is illustrated with existing Late Quaternary (0–18,000 yr B.P.) lake-level records from North America, but the method could be adapted to a variety of types of records, for example sea-level changes or ice accumulation rates. Use of multidimensional scaling is also of more general interest because it puts the duty of devising a suitable dissimilarity coefficient where it properly belongs, on those who understand the nature of the data best.

CHARACTERISTICS OF THE DATA

The data consist of lake-level records from 65 sites in North America (Fig. 1, Table 1). They are a subset of a global data base, known as the Oxford Lake Level Data Bank, which contains records from basins which have been closed during at least part of their Late Quaternary (0–30,000 yr B.P.) history. Closed lakes are used because they are sensitive to changes in precipitation and evaporation, and because short-term variations in these climatic variables can cause relatively large changes in water level. Only sites with a chronology based on radiocarbon or dated tephra layers are included in the North American subset.

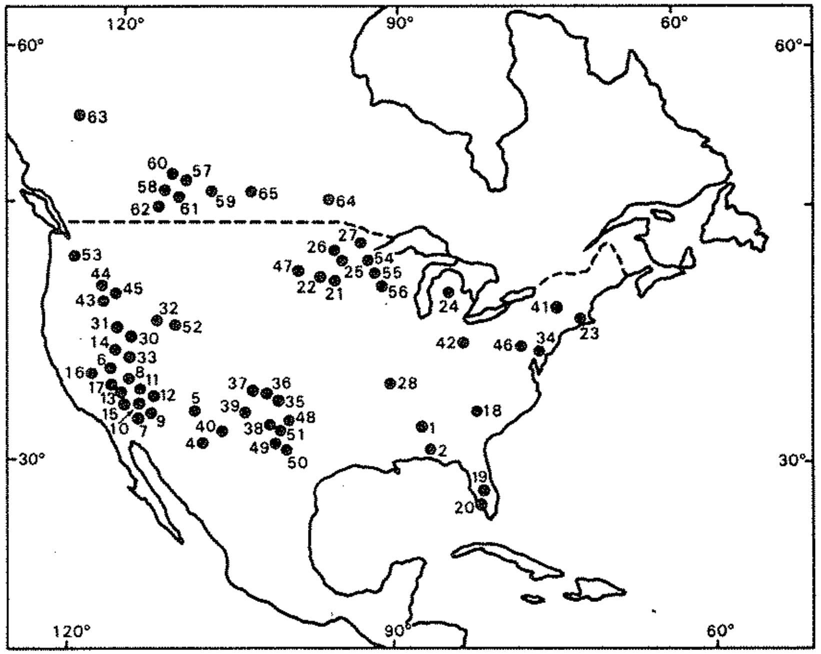

Fig. 1. Distribution of North American lakes used in this study. Identifying numerals refer to Table 1.

Various types of evidence are used to reconstruct past lake levels. Continuous records of lake level changes are derived from cores or exposures of lake deposits, whereas features such as shorelines, algal stromatolites, or lakeside archeological sites yield discontinuous or fragmentary records of changes. Two sorts of information about lake level at a particular time may be inferred from these types of evidence: depth of the lake and direction of change in level. In the Data Bank, these are expressed by *lake status*, an index of relative water depth, and *lake trend*, the direction of change in lake level. These nonstandard definitions of status and trend are retained here because they have been used in the literature based on the Data Bank (Street and Grove, 1976, 1979; Street-Perrott and Roberts, 1983; Harrison et al., 1984; Street-Perrott and Harrison, 1984, 1985; Street-Perrott, Roberts, and Metcalfe, 1985; Harrison and Metcalfe, 1985a,b).

Lakes have different potential and actual ranges in water level. The data are therefore standardized so that records from different basins can be compared. The total estimated range of water depth registered within each basin

Table 1. North American Data Subset

	Basin	Latitude, °N	Longitude, °W	Altitude, (m)
1	Cahaba Pond, Alabama	33.50	86.53	210
2	Goshen Springs, Alabama	31.72	86.13	105
3	Imuruk, Alaska	65.58	163.25	340
4	Cochise, Arizona	32.13	109.85	1260
5	Laguna Salada, Arizona	34.35	110.28	1920
6	Adobe, California	37.91	118.60	1951
7	Clark, California	33.33	116.30	169
8	Deep Spring, California	37.28	118.03	1499
9	Leconte, California	33.33	116.00	−71
10	Manix, California	35.05	116.70	130
11	Manly, California	36.00	116.80	−86
12	Mohave, California	35.37	116.13	276
13	Panamint, California	36.30	117.30	317
14	Russell, California	38.05	118.77	1951
15	Searles, California	35.60	117.70	493
16	Tahoe, California/Nevada	38.93	120.07	1897
17	Tulare, California	36.00	119.67	57
18	White Pond, South Carolina	34.16	80.76	90
19	Annie, Florida	27.30	81.40	36
20	Little Salt Spring, Florida	27.00	82.17	5
21	Kettle Hole Lake, Iowa	43.00	95.00	350
22	Okoboji, Iowa	43.33	95.20	425
23	Duck Pond, Massachusetts	41.93	70.00	3
24	Wintergreen, Michigan	42.40	85.38	271
25	Kirchner Marsh, Minnesota	44.83	92.77	275
26	Rutz, Minnesota	44.87	93.87	314
27	Weber, Minnesota	47.47	91.65	559
28	Old Field Swamp, Missouri	37.12	89.83	97
29	Rosebud, Nebraska	43.00	101.00	380
30	Dixie, Nevada	39.91	118.00	1027
31	Lahontan, Nevada	40.00	119.50	1054
32	Ruby Marshes, Nevada	40.58	115.33	1818
33	Teel, Nevada	38.21	118.34	1495
34	Szabo Pond, New Jersey	40.40	74.48	152
35	Arch, New Mexico	34.08	103.13	1174
36	Blackwater, New Mexico	34.25	103.33	1250
37	Estancia, New Mexico	34.60	105.60	1842
38	Lea County, New Mexico	33.45	103.16	1189
39	Portales Valley, New Mexico	34.44	103.83	1177
40	San Agustin, New Mexico	33.83	108.17	1842
41	George, New York	43.52	73.65	96
42	Browns Lake, Ohio	40.68	82.07	340
43	Chewaucan, Oregon	42.67	120.50	1296
44	Fort Rock, Oregon	43.17	120.75	1311
45	Harney, Oregon	43.20	119.10	1246
46	Longswamp, Pennsylvania	40.48	75.67	192
47	Pickerel, South Dakota	43.50	97.33	395
48	Lubbock, Texas	33.63	101.90	975
49	Monahans Dunes, Texas	31.62	102.77	823
50	Mound, Texas	33.08	102.08	960
51	Rich, Texas	33.28	102.20	1006
52	Bonneville, Utah	40.50	103.00	1280
53	Carp, Washington	45.92	122.88	714
54	Hook Lake Bog, Wisconsin	42.95	89.33	260
55	Mendota, Wisconsin	43.10	89.42	257
56	Washburn Bog, Wisconsin	43.53	89.65	248
57	Hastings, Alberta	53.50	113.00	735
58	Isle, Alberta	52.62	114.43	700
59	Moore, Alberta	53.00	110.50	500
60	Smallboy, Alberta	53.58	114.13	762
61	Wabamun, Alberta	50.54	114.42	723
62	Wedge, Alberta	50.87	115.17	1500
63	Fiddler's Pond, B.C.	56.25	120.75	630
64	Manitoba, Manitoba	51.00	98.00	248
65	Waldsea, Saskatchewan	52.28	105.20	530

Table 1. Continued

during its Late Quaternary history is divided into three status classes (Street and Grove, 1976) as follows:

low (L)	0–15% of the actual altitudinal range of lake depths, including dry lakes;
intermediate (I)	15–70% of the actual altitudinal range;
high (H)	70–100% of the actual altitudinal range, including overflowing lakes.

These categories were originally defined so that each status class had a similar frequency of occurrence in the global data set (Street and Grove, 1979). By chance, this is still true for the North American subset when records for the Late Quaternary (30,000–0 yr B.P.) and for the period since the Last Glacial Maximum (18,000–0 yr B.P.) are considered, but it is not generally true over shorter intervals.

Lake trend is an independent index of the direction of change in lake levels. It may be rising (R), stable (S), or falling (F). These categories are natural, unlike the artificially defined status categories.

Both status and trend are coded at 1000-yr intervals from 30,000 to 0 yr B.P. The coding of continuous records presents no difficulty. Information from discontinuous records is coded only if the critical 1000-yr mark falls within the error limits of the associated radiocarbon date(s). Values of lake status or trend were not interpolated from discontinuous records. The data can be regarded as a series of "snapshot" pictures of lake behavior at specific times for all sites for which information is available.

The current version of the Data Bank (1 Dec 85) includes 68 basins from North America. The record for the North American subset is incomplete, however, with 63% of all possible Late Quaternary status records missing. Data for this study are limited to records from the last 18,000 yr. During this interval, three lakes have no status records and are therefore omitted. For the remaining 65 lakes (Table 1), 50% of status records and 27% of trend records are known (Table 2). Variability from lake to lake is not shown. Over the last 18,000 yr, 8 lakes have a complete status record, but 15 have only three or four records; no lake has a complete trend record (two have 18 and four have 17 records) and 24 have no trend records at all.

For the Holocene (0–10,000 yr B.P.), 64% of status records and 33% of trends are known; during the late glacial (11,000–18,000 yr B.P.) only 30% of status and 19% of trend records are known (Fig. 2). Any patterns which emerge from a consideration of records of the last 18,000 yr are therefore more likely to be affected by what happened during the Holocene than the late glacial, but the imbalance in length of interval and completeness of record is not so large as to invalidate an analysis based on the last 18,000 yr.

METHOD

General Principle

(1) Status categories are defined so that each is approximately equally frequent in the data as a whole. This is far from true, however, for many individual lakes. Some lakes are much less variable than others, partly because of inherent hydrological characteristics. In the same fluctuating climate, one lake may change from L to H whereas another does not move from I. The dissimilarity coefficient (DC) therefore should be based on evidence of change in status rather than on absolute status. This decision has the advantage that change in status has the same physical nature as trend, so information of both types can be combined.

Table 2. Distribution of Lake Status and Trend Classes in the 65 Lakes of the North American Data Subset

Status	Cases (%)	Trend	Cases (%)
0–18,000 yr B.P.			
High	189 (15)	Rising	57 (5)
Intermediate	194 (16)	Stable	227 (18)
Low	237 (19)	Falling	53 (4)
Unknown	615 (50)	Unknown	898 (73)
0–10,000 yr B.P.			
High	103 (14)	Rising	39 (5)
Intermediate	141 (20)	Stable	167 (23)
Low	217 (30)	Falling	34 (5)
Unknown	254 (36)	Unknown	475 (66)

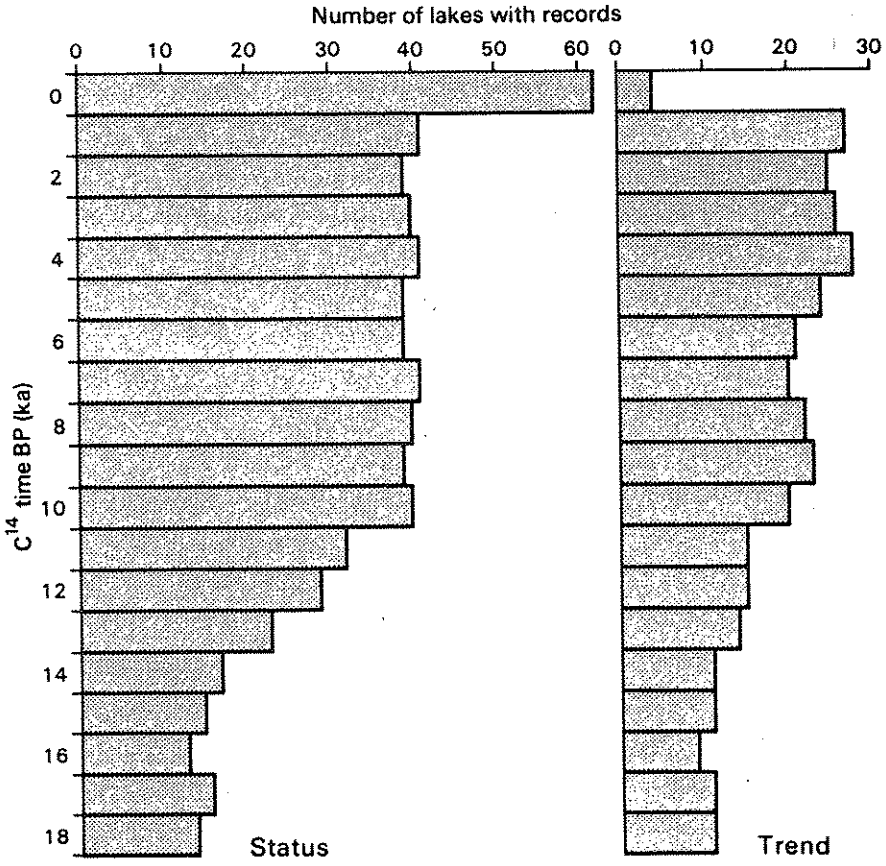

Fig. 2. Distribution of lake status and trend data through time.

(2) Two lakes can be compared only for intervals bounded by times for which a record for both lakes exists, as in the following

	Time									
	1	2	3	4	5	6	7	8	9	. . .
Lake A	H		I	L	L			I		
Lake B			L	L				I		

Here, comparison between lakes A and B is possible for two intervals, between times 3 and 4 and between times 4 and 8. (The identification of intervals for which comparison is possible is a logical AND operation.)

The use of interpolation was considered and rejected because of its unreliability and bias. An analysis of cases in which intervening values were known showed that the accuracy of interpolation of a single value between two known values, even in adjacent time periods, was small ($P \simeq 0.5$). Use of probabilities from the whole subset is undesirable in any case because it imposes the more complete patterns on the incomplete ones.

(3) Consider the example of three lakes with the following records

Lake C	H	H
Lake D	I	H
Lake E	I	H

Lakes D and E have, within the limits of what is recorded, changed identically. Lake C has behaved somewhat differently from lakes D and E, but this may be merely because once it is at H it cannot go any higher. This difference should be reflected by giving greater weight to the value of the DC between D and E than to those between C and D and between C and E.

(4) A DC value based on many common intervals should carry more weight than a DC value based on one or a few intervals only.

(5) A similar change in two lakes between consecutive times should be given more weight than the same change two or more times apart, as in the following example

	Time									
	1	2	3	4	5	6	7	8	9	. . .
Lake F	L	L	I	I	I	I	H	H	H	
Lake G		L						H	H	
Lake H	L	L	I							

Lakes G and H behave identically to lake F, within the limits of the record, so the FG and FH DC values should be minimal but the weight for FG should be smaller than that for FH because one of the comparisons is over six time intervals, over which widely different temporal patterns of change might have occurred. This is not the same point as (4), which reduced the weight of FG and FH equally.

(6) Lakes I and J may behave identically for half their common intervals and differently for the other half. This could give the same DC value as that for lakes K and L, which behaved fairly similarly for the whole time. A similar problem arises with cases in which one lake "tracks" the other. Any individual DC value has little information, but where there are many pairs of lakes for which a DC value can be calculated, multidimensional scaling can separate the lakes in an informative way by placing them according to their pattern of dissimilarities with all of the other lakes.

The Dissimilarity Coefficient and Its Weight

Let the similarity of lakes U and V be S_{uv}, with weight W_{uv}. Let the number of time periods for which the DC is needed be m. Two independent contributions to S_{uv} are defined, one from changes in lake status (L_{uv}), the other from trends (T_{uv}).

Contribution from Changes in Lake Status

Lake status may be high, intermediate, or low (H, I, L). Comparisons are accepted only for those time periods in which both lakes have a status record and changes within each lake are considered only between these times. The maximum number of such comparisons for any one pair of lakes is $m - 1$. Values for the number of comparisons n can range from 0 to $m - 1$. First consider one lake alone. A categoric value can be defined for any given change in status as follows

	Values of a From:		
	H	I	L
To:			
H	0	1	2
I	−1	0	1
L	−2	−1	0

The expected relative frequencies of the five possible values (−2, −1, 0, 1, 2) in random data, assuming equal probabilities of the three status categories, are 1, 2, 3, 2, 1, respectively.

Now consider the comparison of the two lakes. Four possibilities exist:

	Sign	Relative freqeuncy in random data
lakes dissimilar, both change	−	18
lakes dissimilar, only one changes	−	36
lakes similar, neither changes	+	9
lakes similar, both change	+	18

This table defines signs of contributions to similarity and their expected relative frequencies in random data. The magnitude of contributions to similarity can be defined as the sum of categoric magnitudes of change in both lakes plus one. The 1 is added so that any calculated value will be nonzero. This definition, combined with signs as defined above, gives the following matrix of algebraic values of similarity

$$b_{uv} = \begin{array}{c|cc|c|cc} a_u & -2 & -1 & 0 & +1 & +2 \\ a_v & & & & & \\ +2 & -5 & -4 & -3 & +4 & +5 \\ +1 & -4 & -3 & -2 & +3 & +4 \\ \hline 0 & -3 & -2 & +1 & -2 & -3 \\ \hline -1 & +4 & +3 & -2 & -3 & -4 \\ -2 & +5 & +4 & -3 & -4 & -5 \end{array}$$

The expected value of b_{uv} in random data is

$$b_{uv} = [18*(-4) + 36*(-2.5) + 9*(1) + 18*(4)]/81 = -1.0$$

The total contribution of changes in status to similarity is defined as

$$l_{uv} = \left[\left(\sum_1^n b\right)\Big/ n + 5\right]\Big/(2*5)$$

($2*5$ specifies the range -5 to $+5$ in b_{uv}).

Scaling factors in this expression ensure that l_{uv} has the range 0 to 1. The value of l_{uv} is centered on the value expected for random data by

$$L_{uv} = l_{uv} + [1.0/(2*5)]$$

Weights for changes in a single lake are defined

	Values for C		
	From:		
	H	I	L
To:			
H	1	2	3
I	2	2	2
L	3	2	1

thus giving greatest weight to largest changes, and least weight to cases where lake level merely remains high or low. The total contribution of status to weight is then defined as

$$g = \sum_1^n C_u C_v/\Delta$$

where Δ is the number of time periods over which the change is recorded. Values for g range from 0 to $3(m - 1)$.

Contribution of Trend

The trend of a lake at a given time may be rising, stable, or falling (R, S, F). Comparison is possible for any time at which a trend is recorded for both lakes, so the maximum number of comparisons is m. Let there be p such time periods. We define

	Values of t		
	From:		
	R	S	F
To:			
R	1	0	−1
S	0	1	0
F	−1	0	1

and $h_{uv} = (\Sigma_1^n t)/p$ with weight p, which can range from 0 to m. The range of h is from -1 to $+1$. By considerations similar to those for the status measures, the mean value for random data is expected to be $1/9$. Again, scaling to the range (0 to 1) and centering on the expected value gives

$$T_{uv} = (h_{uv} + 1)/2 - 1/18$$

Combination of Status and Trend Information

Contributions to S_{uv} from status and trend are independent, scaled to equal ranges, and are of the same physical nature. They can reasonably be combined using Pythagoras' theorem, but one problem remains. In cases where, for example, many status records but only one trend are known, the value of S_{uv} might be greatly affected by the single trend whereas the large number of status records gave the measure great weight. This is really a problem of small samples and internal weighting within the similarity measure. To circumvent the problem, a factor f between 0 and 1 is used, which rises steeply with the number of items j contributing to the calculation

$$f_j = 1 - 2/(3j^2)$$

The following table shows the behavior of f_j with increasing j

$j = 1$	2	3	4	5	→	∞
$f = 0.33$	0.83	0.93	0.96	0.97	→	1.0

The factor is applied to contributions to similarity from both status and trend. It has the effect of pushing the coefficient toward a central (neutral) value if only one or two records contribute to it. This adjustment reduces the possible excessive effect of a single "wrong" record, at the expense of biasing "correct" single records toward the neutral value (0.5).

Total similarity is

$$S_{uv}^2 = \left[0.5 + f_n(L_{uv} - 0.5)\right]^2 + \left[0.5 + f_p(T_{uv} - 0.5)\right]^2$$

with weight

$$W_{uv} = (g + p)/\max$$

where $\max = 3(m - 1) + m$. S_{uv} ranges from 0 to 1. For convenience, S_{uv} is converted to a DC by

$$D_{uv} = 1.0 - S_{uv}$$

Nonmetric Multidimensional Scaling

Nonmetric multidimensional scaling (Kruskal, 1964a) was devised for use in psychometrics where direct estimates of some or all pairwise differences (DCs) between n "subjects" are often available. To represent the information exactly would need $n - 1$ dimensions, in what may be called the original space. The aim of multidimensional scaling is to represent this information as exactly as possible in a "new" space of fewer, typically two to four dimensions. The number of dimensions is chosen first and then the n subjects are given initial arbitrary positions in the new space. The triangular matrix of $n(n - 1)/2$ intersubject distances in the new space can now be calculated. These distances are usually Euclidean distances, although other choices are possible. A graph (the Shepard diagram) may be plotted with the metric distances (in new space) on the x axis and the dissimilarities (in the original space) on the y axis. If the trial positions in the new reduced dimensional space perfectly represent the information in the original space, points on the Shepard diagram will be monotonic, i.e., a line can be drawn through the points that moves upward and to the right.

With nontrivial data sets, in practice, no arrangement of subjects in the new space satisfies this criterion, so a badness-of-fit measure called stress, based on the sum of squares of deviations of the distances (which are metric) from monotonicity with the dissimilarities, is minimized by an algorithm which moves the subjects in the new space to a locally optimal solution. Values of dissimilarities are not used except to establish the rank order. No way has been found to guarantee a global minimum of stress; but if much the same result emerges when several different initial configurations are used, this result may be accepted as close to the global optimum. The method operates on the Shepard diagram, which may contain up to $n(n - 1)/2$ points, but an important practical advantage of the method is that points in the Shepard diagram may be omitted (because the DC is not known or has low weight) provided only that the number of DCs for a particular subject is at least as many as the number of dimensions in new space.

A version of program KYST (named after Kruskal, Young, Shepard, and Torgerson, who contributed to the technique), modified to take a larger data set and to remove the implicit dependence on integer and real variables having the same word length, was used. This program allows weights to be specified and does not need a complete triangular matrix of DCs. The lake data allowed between $\frac{1}{3}$ and $\frac{2}{3}$ of the DCs to be calculated, depending on the time span used. The most helpful results were produced using the SPLIT = BYROWS option. This option seeks the least bad fit for each lake individually and is equivalent to local scaling (Sibson, 1972). Non-metric scaling is also combined with third-degree polynomial scaling, which is a metric method. This hybrid technique is recommended by Kruskal and Wish (1978) as a means for reducing the risk of settling in local minima that are not the global minimum. The stress decreased rapidly up to three dimensions, but was not much lower in four than in three. Results of one of several three-dimensional solutions with near identical stress are therefore presented.

Identification of Clusters

Groups of lakes showing similar behavior patterns were identified on the basis of their proximity in the three-dimensional space obtained by multidimensional scaling. A variety of conventional clustering techniques proved unsatisfactory in that each produced at least one "cluster" which seemed to be artificial when the original data were examined. Therefore, the following pragmatic approach to define the groups was used.

(1) Some lakes appeared to have been placed arbitrarily in the ordination space. These proved to be lakes whose DC values with other lakes generally had low weight. Sums of weights in all comparisons for each lake were calculated. The frequency distribution of these values was used to determine a cutoff point. Lakes falling below the cutoff point were excluded from the clustering procedure.

(2) Cluster nuclei were defined as the singly or multiply linked clusters at a selected threshold distance in ordination space. In order to select this threshold distance, a series of three-dimensional plots were examined in which all interpoint links shorter than specified distances were drawn. This first threshold distance was chosen subjectively, so as to maximize internal linkages of clusters

while also ensuring that lakes in each cluster showed visually consistent patterns of behavior.

(3) A second (longer) threshold distance was determined such that lakes at this distance still displayed recognizably similar temporal patterns. Lakes linked to others at this level either (a) were added to cluster nuclei with which they had most links, (b) were added to the group to which they showed greatest visual similarity (if they linked with equal frequency to two cluster nuclei), (c) were considered intermediate (if they linked with about equal frequency to two or more cluster nuclei and displayed a behavior pattern with similarities to each of these groups) or (d), if they linked only to lakes not in any of the original cluster nuclei, formed new clusters.

In order to facilitate the display of linkage relationships between lakes, three-dimensional results were projected onto a two-dimensional plane through the ordination space, although this involves the loss of much spatial information. The number of links and the extent of mutual interlinking within a group (Figs. 4 and 7) are important.

RESULTS

For lake-level data over the last 18,000 yr, 24 lakes with a sum of weights less than 4.0 were excluded from the clustering procedure (Fig. 3). The relative position of these lakes varied in different three-dimensional solutions, as would be expected, whereas the relative position of lakes with greater weight varied little. In future analyses, lakes of low total weight should also be omitted from the MDS procedure. Three cluster nuclei were identified at the first linkage level of 0.20 (Fig. 4). At the second linkage level (0.35), several lakes (8, 32, 40, 58) were added to the cluster nuclei; two (44 and 53) were identified as intermediate between nuclei; and a fourth cluster was formed. Twenty lakes were thus allocated to groups. The coherence of the groups thus chosen is shown (Fig. 5). The remaining 21 lakes had unique records, i.e., had no close neighbors (within 35% of the maximum distance between points in the ordination space) and were therefore not allocated to clusters.

The analysis was also performed using Holocene records (0–10,000 yr BP) only. Only 53 lakes had sufficient records to allow comparisons to be made. Fifteen lakes with a sum of weights less than 8.0 were excluded (Fig. 6). Three cluster nuclei were identified at the first linkage level, 0.15 (Fig. 7). Two lakes (19, 58) were added to these nuclei at the second linkage level (0.25) and an additional two lakes (26, 36) were found to be intermediate between nuclei. No new clusters were formed. A total of 17 lakes were included in clusters. The remaining 21 had unique records. The groupings were similar to those found in

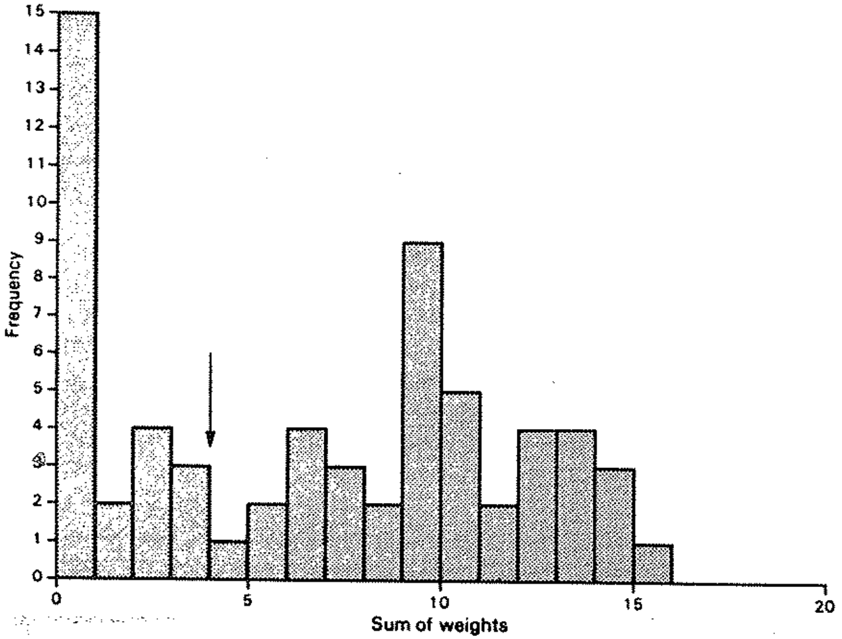

Fig. 3. Distribution of sums of weights associated with lakes in the Late Quaternary analysis. The arrow marks the cutoff point for the clustering procedure.

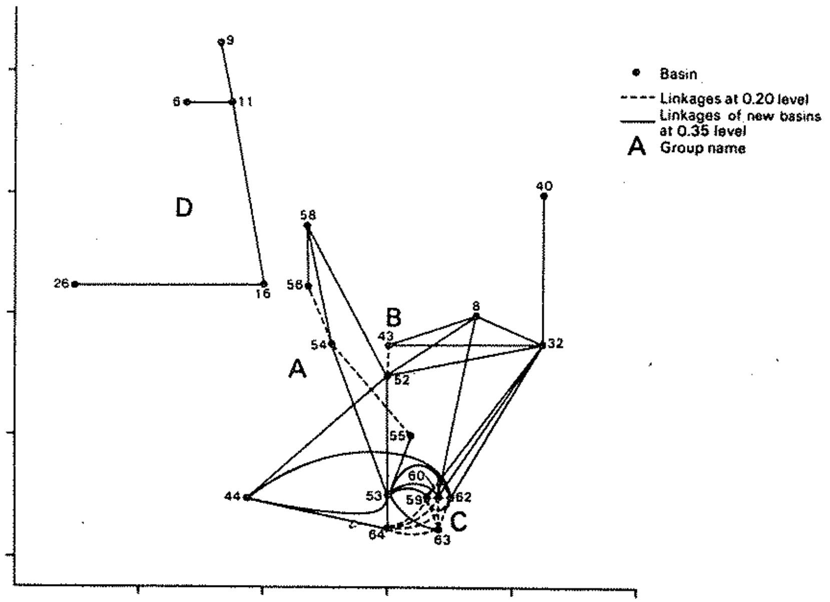

Fig. 4. Linkage diagram of the lakes included in clusters in the Late Quaternary analysis. Points are projected onto a plane through the three-dimensional ordination space. Note that in two dimensions much spatial information is lost. The important features are the number of links and the extent of mutual interlinking within groups A–D.

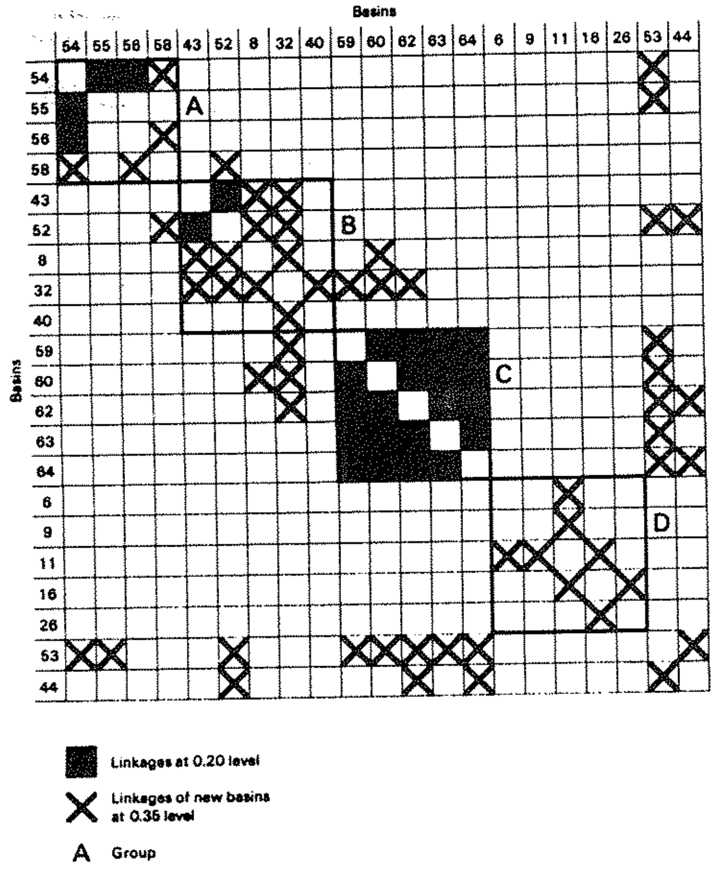

Fig. 5. Matrix diagram showing linkages between lakes included in clusters in the Late Quaternary analysis, to illustrate the extent of coherence of groups thus defined.

the Late Quaternary analysis (Fig. 4, 7). The similarity is partly because there are more data for the Holocene and so the Late Quaternary analysis is weighted toward the Holocene, but it also shows that the patterns identified by the analysis are strong and that the analytical method is robust with respect to a change in the time frame.

DISCUSSION

The four clusters identified in the Late Quaternary analysis (Figs. 4 and 5) show distinctive patterns of lake behavior (Fig. 8). Lakes in Group A were characterized by a mid-Holocene low phase centered on 5000 yr BP, with higher

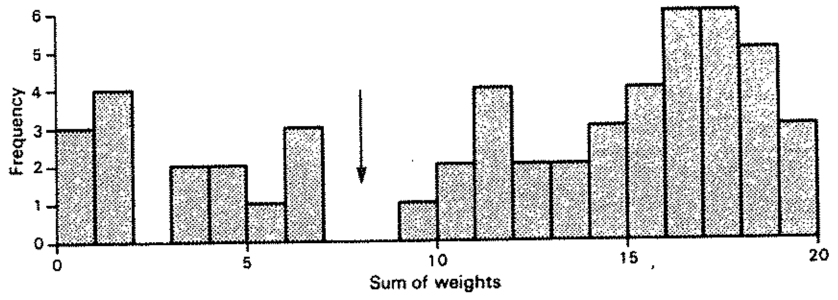

Fig. 6. Distribution of sums of weights associated with lakes in the analysis of Holocene data only. The arrow marks the cutoff point for the clustering procedure.

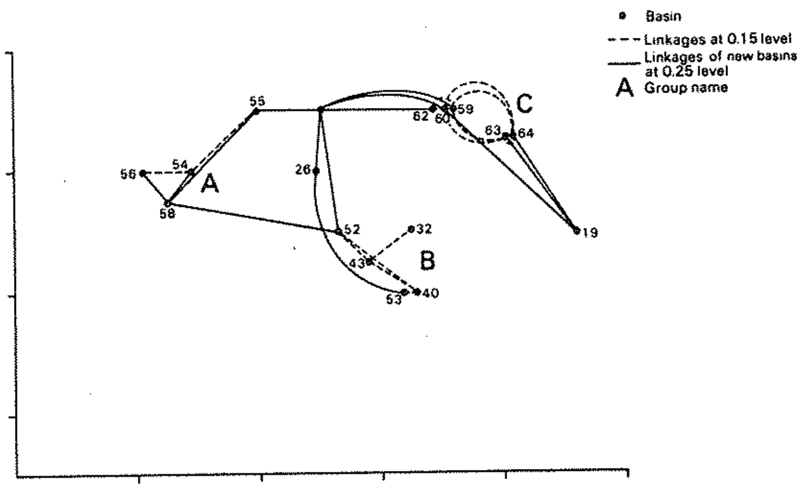

Fig. 7. Linkage diagram of lakes included in clusters in the Holocene analysis, constructed in the same way as Fig. 4.

levels before and after this phase. Lakes with the longest records in this group were highest before 10,000 yr BP. Lakes in Group B were characterized by generally uneventful records. They tended to be relatively high initially, but became low and stable during the Holocene. Lakes in Group C were characterized by an early Holocene low phase, with rising levels in the mid-Holocene, and relatively stable conditions during the last 4000 yr. Lakes in the less tightly linked Group D were initially high but fell at the beginning of the Holocene, with an early to mid-Holocene (ca 9000 to 5000 yr B.P.) low phase, rising levels after 5000 yr B.P., and a return to lower levels toward the present.

With few exceptions, lakes in each group occupy distinct, well-defined geographic regions (Fig. 9). The mean patterns for each group are shown in the

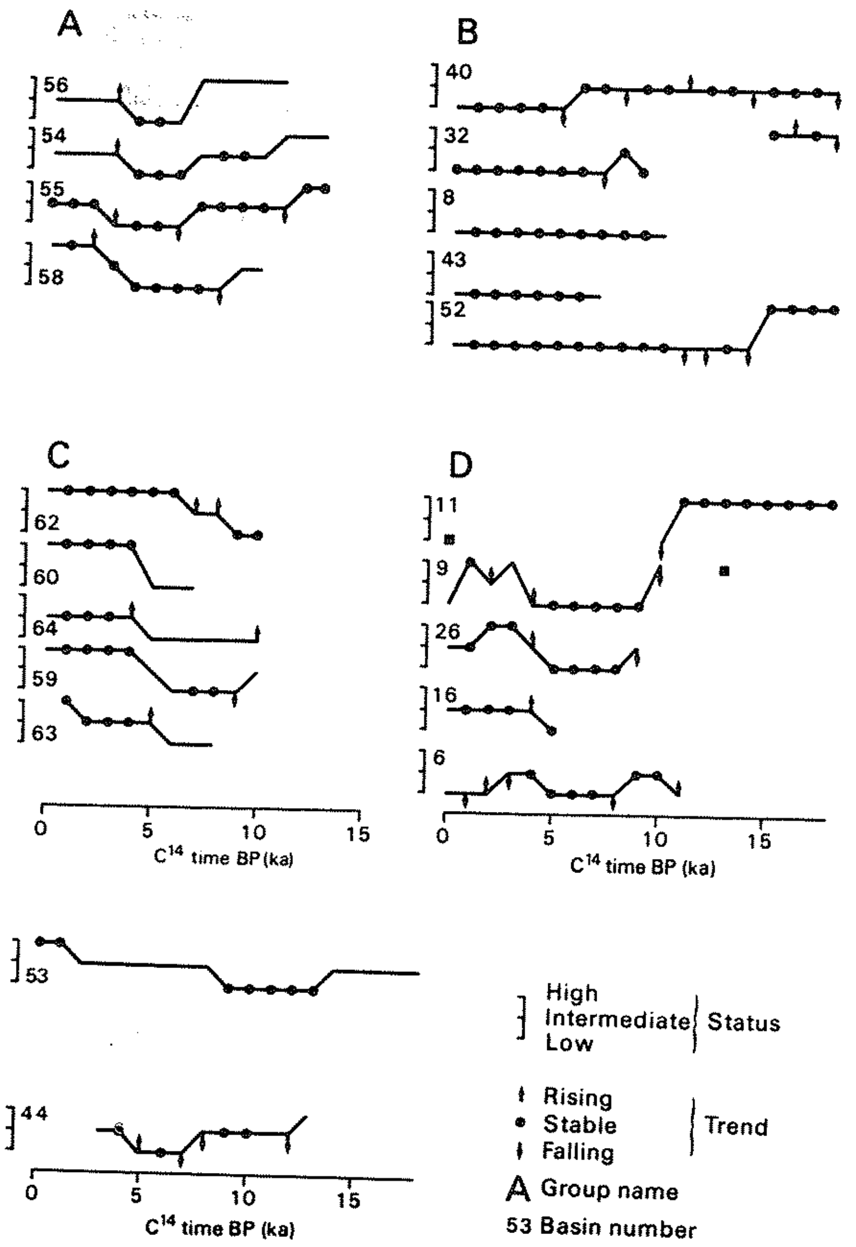

Fig. 8. Patterns of change in lake levels. These are graphical representations of data coded in the Oxford Lake Level Data Bank, rather than continuous lake-level curves. Group names correspond to those in Fig. 4. Lakes 44 and 53 are intermediate between groups.

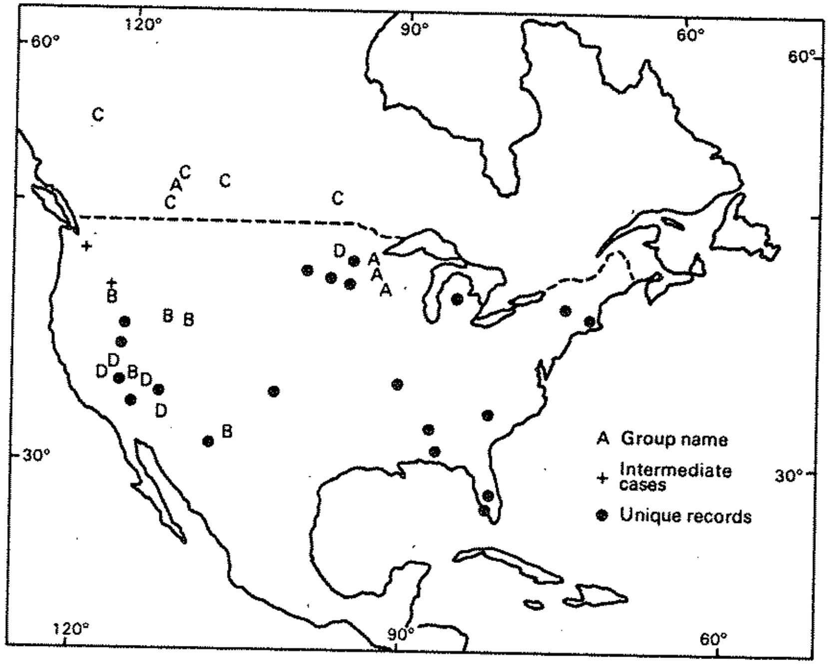

Fig. 9. Spatial distribution of groups distinguished in the Late Quaternary analysis.

form of histograms (Fig. 10) similar to those shown by Harrison and Metcalfe (1985b). Each pattern can be regarded as characteristic for the corresponding region. Thus, high latitude lakes in western Canada, predominantly in Group C, indicate relatively dry conditions during the first half of the Holocene and moister climates during the second half. On the other hand, Wisconsin lakes (Group A) indicate a distinct mid-Holocene arid phase. Such clear-cut differences invite explanations in terms of changing circulation patterns. For example, the climate of both regions is influenced by westerly flow from the Pacific; thus, changes in the strength and position of the westerlies could provide an explanation for the differences between the regions. In the early Holocene, katabatic flow from the remnant Laurentide ice sheet led to suppression of westerly flow of moist Pacific air into the Canadian interior but allowed Arctic air masses to penetrate into the Midwest. As a result, western Canada was dry while the Midwest was wet. After collapse of the ice sheet, westerly flow brought increased precipitation into western Canada but led to replacement of Arctic air masses by warm dry Pacific air over the Midwest (Bartlein et al., 1984; Harrison and Metcalfe, 1985a,b; Kutzbach and Guetter, 1986; Ritchie and Harrison, in press).

Both Late Quaternary and Holocene data sets are characterized by large numbers of lakes with unique records. In many cases, the unique records are

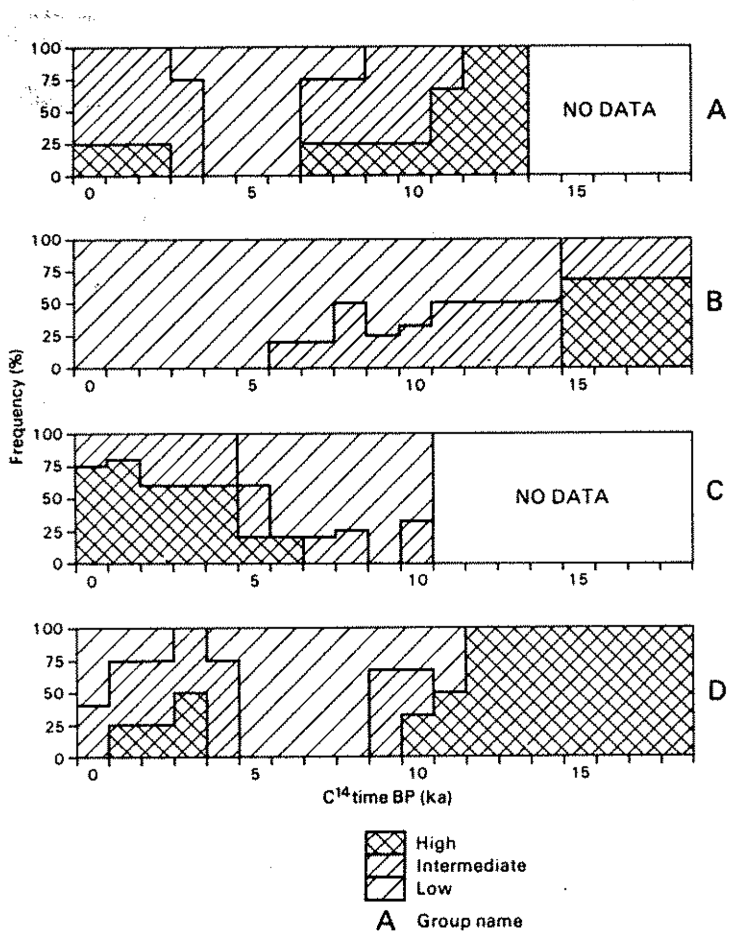

Fig. 10. Histograms showing changes through time in frequency distribution of lake status classes within each group distinguished in the Late Quaternary analysis.

incomplete and have low weight. In the arid southwest, for example, there is insufficient information to distinguish a well-defined regional pattern. Evidence of formerly higher lake levels may have been destroyed, for example by wind erosion, in arid basins (Street-Perrott and Harrison, 1985). In other areas, the lake records are more complete but simply do not resemble one another. This is probably a result of the inherent variability in the response of individual lakes to a given climatic change, caused by differences in their hydrological characteristics. In the southeastern U.S.A., for example, some differences between lake records may be caused by differences in the importance of groundwater inputs to the water budget.

This work has thus not distinguished a consistent pattern in every region because there is not enough similarity among the available records in some regions. Average regional patterns can still be summarized by the histogram technique, but only by imposing some external regionalization (for example, to test a particular hypothesis about differences in the climatic history of two regions). On the other hand, when our analytical method picks out groups of lakes that are spatially coherent as well, average patterns shown by the groups can be regarded as epitomizing hydrological changes in their regions. Differences in patterns shown by different groups constitute strong, internal evidence for spatial patterning in climatic change.

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Notes on the Robustness of the Kriging System[1]

Andras Bardossy[2]

The robustness of the kriging system with respect to uncertainty of the theoretical variogram is investigated. Inequalities for possible changes of the kriging estimator and the estimation variance are derived. Results of a numerical study show that changes of kriging weights can be predicted partly with the help of the maximal kriging weight.

KEY WORDS: kriging, variogram, robustness, estimation variance

INTRODUCTION

The purpose of this paper is to investigate the robustness of the kriging procedure with respect to uncertainty of the estimation of the theoretical variogram. Specifically a measure of the effect of variogram uncertainty is derived, inequality estimates of the change in kriging weights are discussed, an inequality for the change in the estimation variance for a modified nugget effect is derived, and results of a numerical study are presented.

The variogram plays a central role in classical geostatistics. Using ordinary kriging procedure, weights attached to the individual measurements and the estimation variance are calculated from the theoretical variogram and locations of the measurement points. Actual values of measurements enter this procedure at two points: (1) in estimation of the variogram, and (2) in computing the estimator value after computing kriging weights. Several attempts have been made to overcome distributional problems in the estimation of the experimental variogram. Different estimators were proposed for a more robust estimator of the experimental variogram (Cressie and Hawkins, 1980; Dowd, 1984; Omre 1984; etc.). In contrast, few publications are concerned with the effect of possible error in the choice of variogram type and variogram parameters. Journel and Huijbregts (1978, p. 233) give an example showing that kriging results are essentially insensitive to the variogram type, provided the model fits correctly the experimental curve. Diamond and Armstrong (1984) investigated the prob-

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