9. A Model of Peat Bog Growth

R. S. Clymo

From an analysis of the processes involved in peat growth, a mathematical model is constructed and used to predict bulk density and age profiles which are compared with observed ones. Given only bulk density and age profiles of the peat, it is possible, alternatively, to estimate the model parameters of growth, decay, compression, and consolidation.

9.1 Introduction

One recent estimate (Tibbetts, undated) is that peat covers 150×10^6 ha—between 1 and 2% of the total land surface of the earth. From estimates given by Olenin (1968) one may calculate that there are about 200×10^9 t of peat. This is the same order of magnitude as the estimated total dry matter production on Earth in one year (Fogg, 1958). In brief, there is a lot of peat, and the dynamics of peat growth would be worth studying for this reason alone.

Peat deposits are not uniform, either in structure or in rate of growth (Walker and Walker, 1961; Walker, 1970). Any attempt to account for these differences must involve some sort of conceptual model of the processes involved. At present these models are mostly qualitative. For example, Walker (1970), who is more explicit about the processes involved than most workers, writes: "... sedentary deposits are modified after their initial deposition, chemically by humification and physically by compaction. ... A rising water table leaves little time for extensive humification and ensures the rapid accumulation of little altered plant remains. Periodically or permanently dry surfaces, however, allow the dry oxidative breakdown of dead plants ..."

"In most ... mires dry oxidation is limited and humification virtually ceases as the newly deposited material sinks into the anaerobic zone. There the accumulation of more material above leads to some compaction ... compaction is soon almost totally accomplished and increases very little as more deposit accumulates above. However, compaction is the inevitable result of the removal of water from a deposit ...". Kaye and Barghoorn (1964) discuss the same problems of autocompaction for salt-marsh peat with a bulk density about seven times that of bog peat. They give a sketch graph of the way in which a given level in the peat moves with time.

One may hope that further understanding will follow the making of more precise quantitative models since the assumptions become more clear, and testable predictions can be made. Gore and Olson (1967), following Jenny et al. (1949) and Olson (1963), have made a simple model, in which dry matter was added by production, lost by decay, and moved between notional compartments. Jones and Gore (Chap. 8) have elaborated this model to allow decay rates varying with depth.

This paper describes a more complicated model, involving both dry matter and peat depth. In part 9.2.1 the processes involved in peat growth are described, functional relationships examined, and the model constructed. In part 9.2.2 data for testing the model are described, and the model is used to make predictions which are compared with measurements.

9.2 The Model

Parameters of the model

Symbol	Meaning	Dimensions	Units used in this paper
p	Net productivity	$ML^{-2}T^{-1}$	$g\ cm^{-2}\ yr^{-1}$
L	Rate of growth in length of plants at the bog surface	LT^{-1}	$cm\ yr^{-1}$
α_1	Aerobic decay constant (at depths above *W*)	T^{-1}	yr^{-1}
α_2	Anaerobic decay constant (at depths below *W*)	T^{-1}	yr^{-1}
k	Compression constant	$M^{-1}LT^2$ (1/stress)	$g\ (f)^{-1}\ cm^2$
c	Creep constant	(log time cycle)$^{-1}$	$(\log_{10}\ \text{cycle yr})^{-1}$
W	Water table depth below surface	L	cm
p'	Rate of addition of dry matter to the anaerobic zone (below *W*)	$ML^{-2}T^{-1}$	$g\ cm^{-2}\ yr^{-1}$
L'	Rate of depth addition to the anaerobic zone (below *W*)	LT^{-1}	$cm\ yr^{-1}$

These parameters are referred to later by symbol only.

9.2.1 Making the Model

There are many types of peat-forming system, and many systems of classifying them; examples are given in Robertson (1968), Anon (undated). The general types considered here are sedentary (autochthonous)–principally oligotrophic telmatic and terrestrial systems (West, 1968). It is helpful to use the specific case of a pure carpet of a vertically growing species of *Sphagnum*. Rate of vertical increment to the surface is then easily visualised.

The rate of at least some processes involved in peat accumulation is not constant. Dry matter and length increment, for example, show diurnal and seasonal fluctuations. In this model, which is concerned with times measured in tens or hundreds of years, mean annual rates are equated with instantaneous rates: processes with a short time constant are assumed to have no effect on those with long time constants, in the same way that a moving coil galvanometer will register a drift in a DC voltage but give no apparent response to a superimposed 50 Hz alternating voltage. Forrester (1961) shows that this assumption may be untrue in systems with interacting feedback loops.

The model includes dry matter (affected by productivity and decay), depth (affected by rate of growth of plants, decay, compression and secondary consolidation) and depth of the water table. Functional relationships of these quantities are now considered.

9.2.1.1 Dry Matter

It is assumed that dry matter is added to the surface at a constant rate per unit area which may be equated with the net annual productivity p (dimensions $ML^{-2}T^{-1}$). The credibility of this assumption is reduced as time increases, since the major determinants of productivity (climate, nutrition, water supply) are all likely to change. Even if climate and nutritional conditions remain constant, it seems doubtful if the bog water table will continue to rise at the same rate as the bog surface (Granlund, 1932) and the productivity of *Sphagnum* is known to be much affected by the depth of the water table (Clymo, 1970).

It seems to be generally agreed that some decay (equated with loss of dry matter) occurs, and measures of this for *Sphagnum* are given by Clymo (1965). Oxidative breakdown results in transformation of dry matter to soluble or gaseous forms. In many peat bogs the hydraulic conductance for lateral flow in the top 20 cm is relatively great (Romanov, 1961; Chapman, 1965), though the conductance of peat at lower depths is much reduced (Boelter, 1965; Ingram et al., 1974). Both soluble and gaseous material may then be removed in the surface layers, though it may be that loss as gas is the major component at depth. Humification, i.e. the loss of plant structure, often accompanied by darkening of colour, is included within the definition of decay, insofar as it results in loss of dry matter from the system.

There is little direct evidence extending over more than a few years about the form of the functional relationship between decay rate and amount of material. Baker (1972) measured the weight of 1 cm sections of stems of *Chorisodontium aciphyllum*, which forms peat banks at least 170 cm deep on Signy Island (lat. 60° 43′S, long. 45° 38′W). He concluded that, ignoring compression if it occurred, the decay rate was a linear function of time, at least to a depth of 10 cm (corresponding to an age of about 30 years). Heal, Latter, and Howson (Chap. 7) report weight losses of plant material in nylon mesh bags at Moor House. Some of their results, and those of Baker, are shown in Figure 1a. These results could be described by:

$$x = x_o - \alpha' t \tag{1}$$

where x is the amount (per unit area) at time t, x_0 is the amount (per unit area) at $t = 0$, α' is a decay constant (dimensions $ML^{-2}T^{-1}$).

The difficulty comes when one tries to extrapolate these data to longer times. If Baker's linear regression is followed, his plants should have entirely decayed at 14.5 cm deep (about 45 years), but the peat bank is 170 cm deep and C-14 dating shows it to be about 1800 years old.

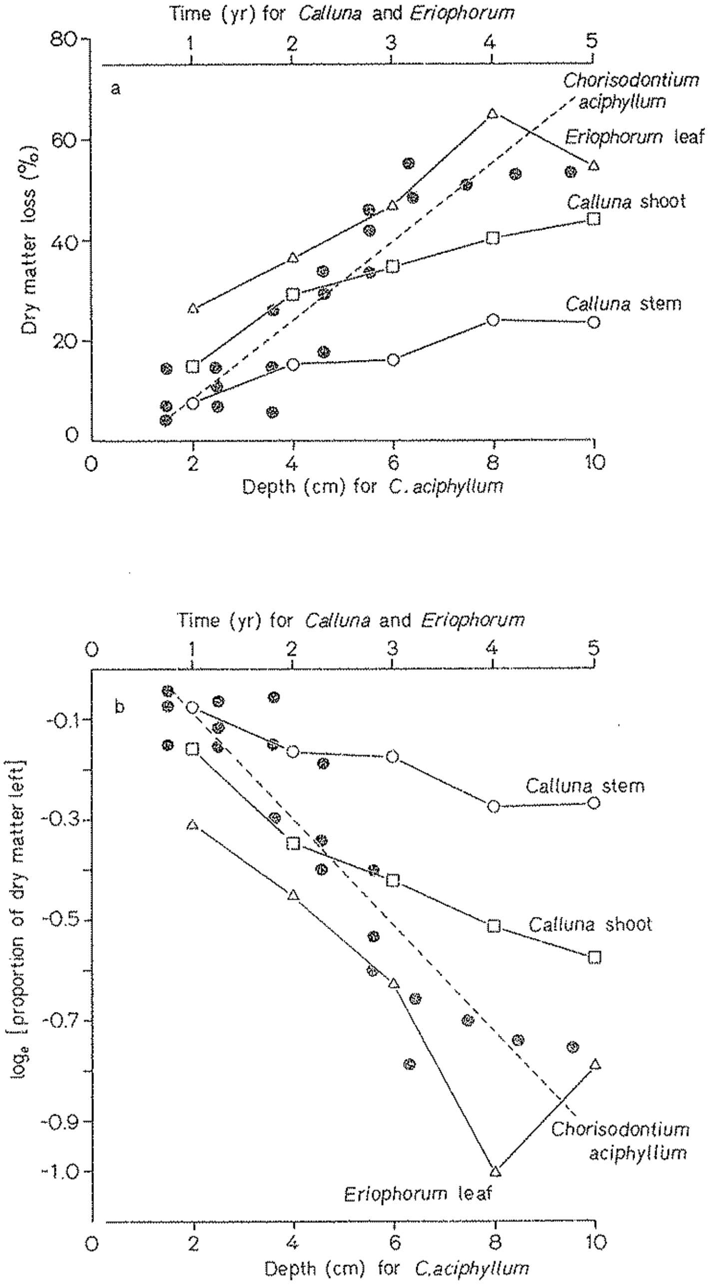


Fig. 1 a and b. Dry matter loss from four materials as a function of time. *Calluna* shoot, *Calluna* stem, and *Eriophorum* leaves: (from Chap. 7). *Chorisodontium*: ● (from Baker, 1972). *Dashed line* is the linear regression to the *Chorisodontium* points. (a) graph plotted on linear axes; (b) graph uses the same data but the dependent variable axis is logarithmic

A commoner, and more plausible, assumption is that the rate of decay is proportional to the amount of matter at the time:

$$\frac{dx}{dt} = -\alpha x \tag{2}$$

which gives

$$x = x_o e^{-\alpha t} \tag{3}$$

where α, the decay constant, has dimension T^{-1}. If this were a good description one would expect log (x/x_o) to be a linear function of t. The results of Heal, Latter, and Howson and of Baker are plotted in this way in Figure 1b. The fit (Table 1) is almost as good as in Figure 1a.

One might indeed expect the rate of decay to decline faster than this, because there is reason to believe that plant constituents are broken down at differing rates so that the material left after a few years is more resistant to decay. Minderman (1968) has shown this to be likely in a temperate pine forest. In a *Sphagnum* valley bog, *Sphagnum* peat from 60 cm brought to the surface decayed less rapidly than did surface material (Clymo, 1965), and Heal, Latter, and Howson (Chap. 7) have shown that O_2 uptake rate of *Eriophorum vaginatum* leaf litter recovered from the field (with attendant microorganisms) declines by a factor of about two to three over five years. *Calluna* shoot and stem litter did not show this decline, and the results in Figure 1 do not suggest such an effect either. For the model, therefore, the functional relation [Eq.(2)] has been used.

Apart from the effects already mentioned there is direct evidence (Clymo, 1965) that the rate of decay of comparable age *Sphagnum* is lower at depth in the peat than at the surface. Heal, Latter, and Howson (Chap. 7) found that *Calluna* stems and *Juncus effusus* leaves gave similar results (though *E. vaginatum* roots showed a puzzling small increase in decay rate at greater depth). The change in tensile strength of unbleached calico strips also suggests a reduction in decay rate with depth. The major decrease in decay rate was correlated (Clymo, 1965) with the change from aerobic to anaerobic conditions at about the water table. The

Table 1. Fit of decay data to two models

Material	Number of esti-mates	Correlation coefficient		Variance: $\frac{\text{residual}}{\text{total}}$	
		Linear model (1)	Negative exponential (2)	Linear model (1)	Negative exponential (2)
Chorisodontium aciphyllum (whole plants)	20	0.987	−0.917	0.026	0.160
Calluna vulgaris (stems)	5	0.944	−0.911	0.108	0.156
Calluna vulgaris (leaves)	5	0.937	−0.929	0.122	0.138
Eriophorum vaginatum (leaves)	5	0.882	−0.863	0.223	0.256

calico data can be interpreted as approximating such a steplike change. Type and activity of microorganisms change from aerobic peat to anaerobic (Chap. 5), though there is dispute about the viability and activity of microorganisms at depths of 1 m or more in peat (Waksman and Stevens, 1929; Waksman and Purvis, 1932; Burgeff, 1961).

Jones and Gore (Chap. 8) prefer to fit a linear regression with depth to the tensile strength data (Chap. 7), and such a description is certainly preferable for the data on decay of *Calluna* stems and *J. effusus* leaves. They also show that their model is sensitive to the slope of this regression.

As a compromise it has been assumed here that one decay parameter, α_1, is applicable throughout the aerobic zone, and a second parameter, α_2, applies in the anaerobic zone below the water table at depth W. The functional relation [Eq.(2)] is used in both. In this model, sensitivity to small changes in W is equivalent to Jones and Gore's change of decay/depth regression coefficient.

9.2.1.2 Depth

As in the case of dry matter and with the same reservations, it is assumed that the positive rate of growth in depth, L, attributable to upward growth of *Sphagnum*, is constant. For a sward of *Eriophorum* or *Calluna* the concept of upward growth of the plants is less obvious, and L would be a notional parameter not easily equated with any linear measure of leaf or shoot growth.

Removal of material during decay may be expected to reduce depth, just as the removal of a brick near the bottom of a pile will reduce the height of the pile. This simple view is the one adopted here, for lack of direct evidence to the contrary. Where visible structure has been lost this assumption is plausible. Where structure is still present, it seems more likely that decay may not directly reduce the depth, just as the removal of isolated bricks from a wall does not at first reduce the height of the wall (though eventually there is a complete and sudden collapse). Decay may however have a secondary effect by altering the compressive and creep properties. Again, evidence is lacking.

From the assumption that decay affects dry matter and depth in the same way, it follows that bulk density, defined as $\Delta x/\Delta l$ (where l is the depth at time t corresponding to the dry matter per unit area, x) would be constant with depth, although the age profile would depend on α, p and L.

Nevertheless, the bulk density of peat from 50 cm below the surface is commonly 0.1 g cm^{-3}, about 10 times that at the surface. Peat *can* be compacted further: the "Finbloc" of commerce has a bulk density of about 1.25, about 100 times the original value. Jones and Gore (Chap. 8) point out that selective decay of less dense plant materials can of itself lead to increased bulk density. The argument could be extended to different fractions of one plant. Such an effect could not account, however, for the observed increases in bulk density by a factor of ten, particularly for a peat formed almost entirely from *Sphagnum*. It seems then that a model of peat bog growth should attempt to include compaction, however imperfectly.

The general features of peat consolidation have been investigated by engineers, and are discussed by Barden and Berry (undated) and by Berry and Poskitt

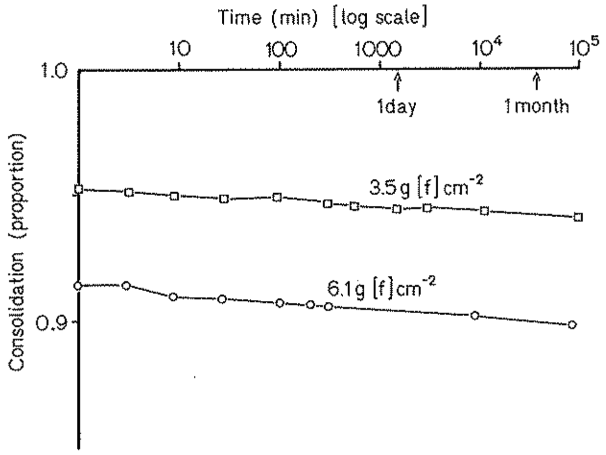


Fig. 2. Consolidation of *S. magellanicum* peat cylinders 21 cm diameter and 5 cm thick under different stresses applied for eight weeks

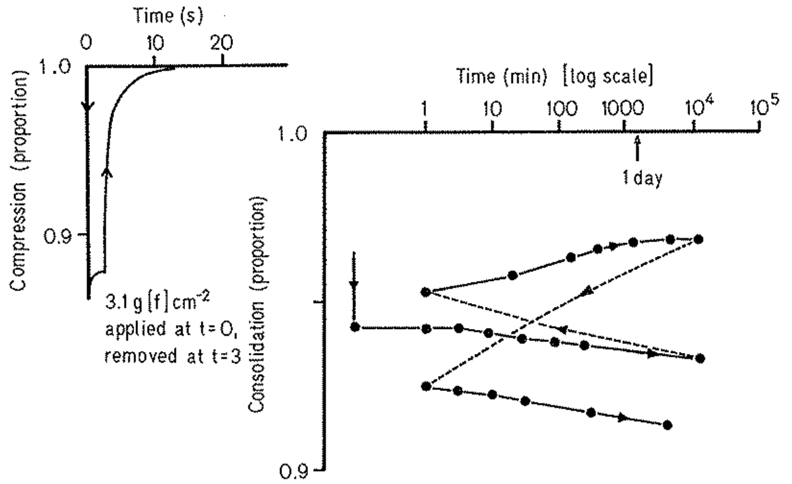


Fig. 3. Consolidation and recovery under repeated application and removal of stress, of cylinders of peat similar to those of Figure 2. *Dashed lines* show rapid (< 10 s) responses when stress was first applied or removed. *Inset:* short term compression and recovery

(1972). They refer to much of the other work in this field. The stresses of interest to engineers are usually an order of magnitude greater than those developed naturally. However, the same general features are shown at stresses of interest to ecologists. Examples of one dimensional strain vs compressive stress curves for peat are shown in Figure 2. They are for discs of *Sphagnum magellanicum* peat, 5 cm thick by 21 cm diameter. The discs were cut from a core, and span the depth 6–11 cm. (The apparatus and method used are described in Sect. 9.5.2).

There were two phases of consolidation, the first with a time constant of a few seconds at the most, and the second which was long continued. The strain during the first phase was nearly reversible for at least 22 cycles if the stress was removed

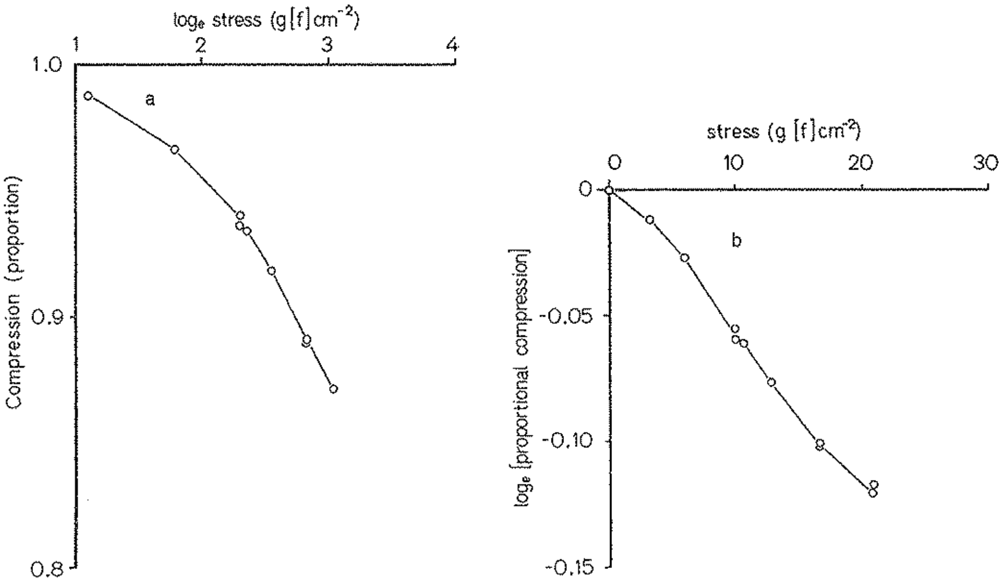


Fig. 4 a and b. Compression under stress of short duration of peat cylinders similar to those in Figure 2. The two graphs show the same data plotted using different axes

rapidly (Fig. 3 inset), and might be called elastic. Barden and Berry postulate that during this primary phase any excess pressure produced in the macrocapillary pores will be dissipated at a rate depending on the thickness of the peat slice. The relation of strain to stress for brief applications is shown in Figure 4. For these large strains (equivalent to stresses produced by about 3 m depth of peat) a linear relation is perhaps not to be expected. Barden and Berry (undated) assume a linear relation between "void ratio" in the peat and log stress. Figure 4a shows data which should give a straight line on this assumption. The fit is not particularly close, and there are difficulties in implementing this relationship in a model where the applied stress starts at zero. For descriptive purposes a negative exponential (Fig. 4b) is also a fairly close approximation at least within the range of interest:

$$\frac{dZ}{d\varepsilon} = -kZ \tag{4}$$

where ε is the stress, Z the slice thickness, and k the compression constant (dimensions $M^{-1}LT^{2}$).

During the second phase (secondary consolidation by creep of the peat skeleton) the strain is approximately proportional to log time.

For "granular amorphous" peat the rate of creep is little affected by the applied stress (Fig. 5). This rather surprising feature is shown in several sets of experimental measurements (Hanrahan, 1954; Barden and Berry, undated; Berry and Poskitt, 1972). The results in Figure 5 indicate that even the fresh *Sphagnum* peat behaved as if it were "amorphous". This behaviour is described by:

$$Z/Z_o = 1 - c_a \log(t + m) \tag{5a}$$

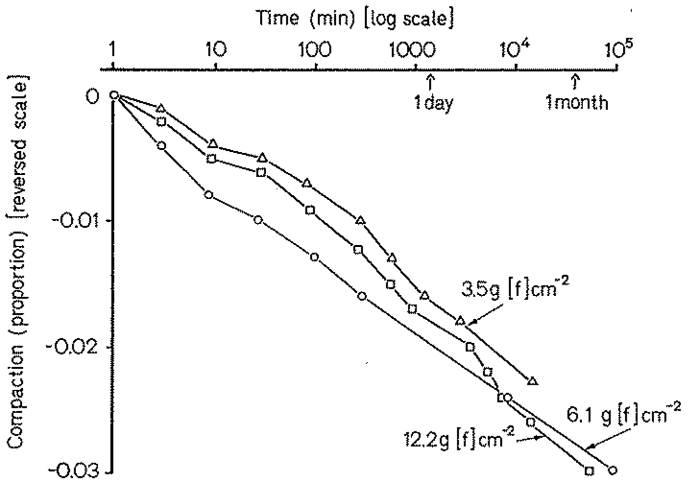


Fig. 5. Secondary consolidation (creep) of cylinders of peat similar to those of Figure 2 under different stress. The lines are arranged to coincide at $t = 1$ min, and the proportional compression is expressed as the proportional decrease from the thickness at 1 min

where Z is the slice thickness at time t, Z_o the initial thickness, c_a is the creep constant for amorphous peat, dimensions (log time cycle)$^{-1}$, and m is an arbitrary number, small compared to the range of t, defining the origin of the log time axis.

The mechanism of creep in these peats is postulated to be the relative movement of separate elements of the peat skeleton, the rate being affected by the thin layer of highly viscous water around each particle.

For "fibrous peat", although the linear strain/log time relationship still holds, the slope depends on the applied stress too. From Barden and Berry's Figure 9a:

$$Z/Z_o = 1 - \varepsilon c_f \log(t+m) \qquad (5\text{b})$$

where c_f is the creep constant for fibrous peat. The rate of creep in this case is thought to be controlled by the slow squeezing out of water from microcapillary spaces inside the peat skeleton.

The existence of two relationships makes model construction awkward, or undesirably arbitrary, since any one peat mass may start behaving according to Equation (5b) but gradually change to Equation (5a). For tests, both Equation (5a) and Equation (5b) were used on one core, and the results compared.

It seems to be generally agreed that the rate of the first phase depends on sample dimensions—presumably because the dissipation of pore pressure occurs as a result of flow of water out of the sample, and the longer the path the greater the resistance. Hanrahan (1954) finds the same "scaling" effect for the second phase too, but Berry and Poskitt (1972) were unable to confirm this. In the first phase, even with a 3.5 m slice of peat the response time is less than two months. Since the conditions in a growing peat column are changing so slowly, and since there is doubt about whether the second phase rate is affected by scale at all, no allowance is made in the model for such effects in either phase.

Kaye and Barghoorn (1964) used the relationship between creep and log time, and noted that the rate of upward growth of deep peat may, as a result of creep throughout the mass, become very sensitive to small changes in surface conditions. Even without environmental changes however, it is implicit in Equation (5) that at some value of t the slice of peat vanishes (and at longer times has negative thickness). This is obviously impossible and eventually the relationships of Equation (5) must become poor descriptions. The "vanishing time" is therefore, a statistic of this model which is worth checking. A corollary is that although the model predicts a steady state mass equal to p/α, the bulk density should continue to increase.

If the first phase consolidation were truly elastic, one would expect that on removing the top layers of peat, the lower layers would expand noticeably. This happens only to a very limited extent, possibly because other changes in the peat make the initial elastic change permanent as is shown in Figure 3. It is still possible, however, for peat from some depth to undergo further initially elastic compression if a stress is applied.

The experimental measurements for creep cover a year at most and, as with decay, one is forced to extrapolate grossly. The model also assumes that the compression and creep parameters (k and c) remain constant, which seems unlikely. The decay parameter, α, does now affect the bulk density profile, but it does so indirectly through effects on compressive stress and creep time.

Assembly and implementation of this model are described in Section 9.5.1.

9.2.2 Checking the Model

9.2.2.1 Introduction

The model needs values for the seven parameters $(p, L, \alpha_1, \alpha_2, k, c, W)$ and specified depths below the surface, and predicts values both for age at each specified depth and for bulk density of the layer between depths.

There are two ways of checking the model. Much the simpler is to measure the seven parameters and bulk density profile, and if possible date the profile, then compare the predicted and observed age and bulk density profiles. The model can also be stimulated by varying the parameters and observing its responses. This method of checking will be referred to as the "direct method".

The second, more complicated, method is to provide a bulk density profile and if possible an age profile, and attempt to minimise some function of the difference between observed and predicted profile(s) by adjusting the parameters. This will be called the "indirect method". The results are estimates of all seven parameters and, if no age profile is used, a predicted age profile. These may be compared with such measurements as happen to be available—one does not need all seven as in the first method. Unfortunately the second method costs about 10^4 times as much in computer time/space, and does not produce a unique solution. Some of the problems are discussed by Plinston (1972). For two parameters, one may visualise the problem as equivalent to locating the lowest point on an undulating surface, where altitude represents "badness of fit", and parameter values are given by co-ordinates in a N–S and in an E–W direction.

One attempts to find the "least bad fit". All methods for doing this may settle into any hollow in the surface, which need not be the lowest one. Whether they do so or not depends on starting point and step length. There may be a long flat-bottomed valley, rather than a nearly circular hollow. If this valley is oriented N–S or E–W, the implication is that one of the parameters can vary a great deal without much effect on the "badness of fit", i.e. that the model could do without it. If the valley runs NE–SW (or on the other diagonal) a change in one parameter can be compensated by a change in the second; the parameters are correlated. Again the model gains little from having two separate rather than one composite parameter. The more parameters, the greater the likelihood of correlations of this kind.

Both direct and indirect types of check have been tried.

9.2.2.2 Methods

The methods are described in Section 9.5.2.

9.2.2.3 Results and Discussion

Detailed Investigations on a Single Site. A detailed study was made, by both direct and indirect methods, of cores from a specific site (MH l) 2 m by 1 m on Burnt Hill at Moor House (nat. grid. ref. NY754328). The site is an almost pure *S. magellanicum* lawn with a few small plants of *Calluna vulgaris* and *Eriophorum angustifolium*. Cores were taken from places with no emergent vascular plants.

Peat Characteristics. The seven parameter values measured are shown in Table 2. Values for p and L were measured directly. The aerobic decay rate, α_1, was measured on four similar lawn sites, within 200 m, as integrated carbon dioxide flux using

$$\alpha_1 = -\log(1 - x_f/x_r) \tag{6}$$

where x_f is the integrated flux of carbon, expressed as CH_2O equivalent, and x_r is the total dry matter "at risk"; in this case dry matter in the aerobic zone. If

$$x_f \ll x_r \quad \text{then}$$

$$\alpha_1 \simeq 1 - x_f/x_r\,. \tag{7}$$

The first estimate of α_1 in Table 2 uses this method. The second estimate was obtained from direct measurement of dry weight lost from material in nylon mesh bags at a similar site about 150 m distant, using Equation (3). The third estimate, 0.062 from Al, Ti, and Mg concentrations, is described later.

The estimate of α_2 was derived from integrated methane fluxes during the same time as the carbon dioxide fluxes. A peat depth of 3 m with average bulk density of 0.1 g cm^{-3} was used to estimate x_r for use in Equation (7). The samples for this "long core" were collected in cylinders 7.5 cm diameter by 10 cm long from a peat face exposed in an erosion gulley 250 m from the main core site.

Table 2. Measured parameter values for *Sphagnum magellanicum* lawn

Productivity (p)	0.016 g $\mathrm{cm}^{-2}\ \mathrm{yr}^{-1}$ (0.007, $n=12$)		
Plant length growth rate (L)	1.5 cm yr^{-1} (0.3, $n=12$)		
Aerobic decay (α_1)	0.032 ($n=4$)	0.050 (0.021, $n=40$)	0.062 yr^{-1} ($n=1$)
Anaerobic decay (α_2)	$10^{-5}\ \mathrm{yr}^{-1}$ ($n=4$)		
Primary compression (k)	0.005 $\mathrm{g(f)}^{-1}\ \mathrm{cm}^2$ ($n=3$)		
Secondary creep (c)	0.018 ($\log_{10}$ time cycle)$^{-1}$ ($n=3$)		
Water table (W)	8 cm ($n=3$)		

Figures in parentheses are standard error of estimate and number of observations.

Some check on this estimate of α_2 is possible. If the dry matter part of the model is correct, then from Equation (A2) in appendix, 9.5.1:

$$\alpha_2 < p'/x_b \tag{8}$$

where x_b is the dry matter per unit area accumulated to date, and p' is the dry matter per unit area passing into the anaerobic zone. Since $x_b < x_\infty$, taking $p' = p$ sets an upper limit on α_2. In this case the maximum value is 0.7×10^{-3}, an order of magnitude greater than the measured value.

It is also of interest that Svensson (1973) has found similar fluxes of CO_2 and CH_4 from a subarctic mire near Abisko (Sweden).

Estimates of compression (k) and creep (c) parameters were obtained from the slopes of lines similar to those of Figures 4 and 5. The water table depth (W) was measured (in the holes left when cores were removed) at three times in the following year.

Where possible, standard errors are shown in Table 2. From the nature of the methods, however, it seems likely that systematic error is greater than random error in the estimates of α_2, k, c, and W. In consequence significance tests could be misleading, and are not reported.

Bulk density, β-activity, lead concentration and non-destructive relative Cs-137 estimates were made on one core. Much more reliable absolute Cs-137 estimates were made on a second core about 30 cm from the first. Al, Ti, Fe, Mg, Cu, Cd, Zn, and Pb concentrations were measured on a third core.

The metal concentration and bulk density profiles are shown in Figure 6 and the Cs-137 and β-activity profiles in Figure 15 (Sect. 9.5.3). The two lead profiles were measured by different analysts on different cores. The correlation between them is 0.83. The concentrations of Mg and Fe in the top 5 cm are 860 ppm and 2420 ppm, similar to the values (740 ppm and 2160 ppm) recorded by Gore and Allen (1956) from Moor House blanket bog with *S. rubellum*.

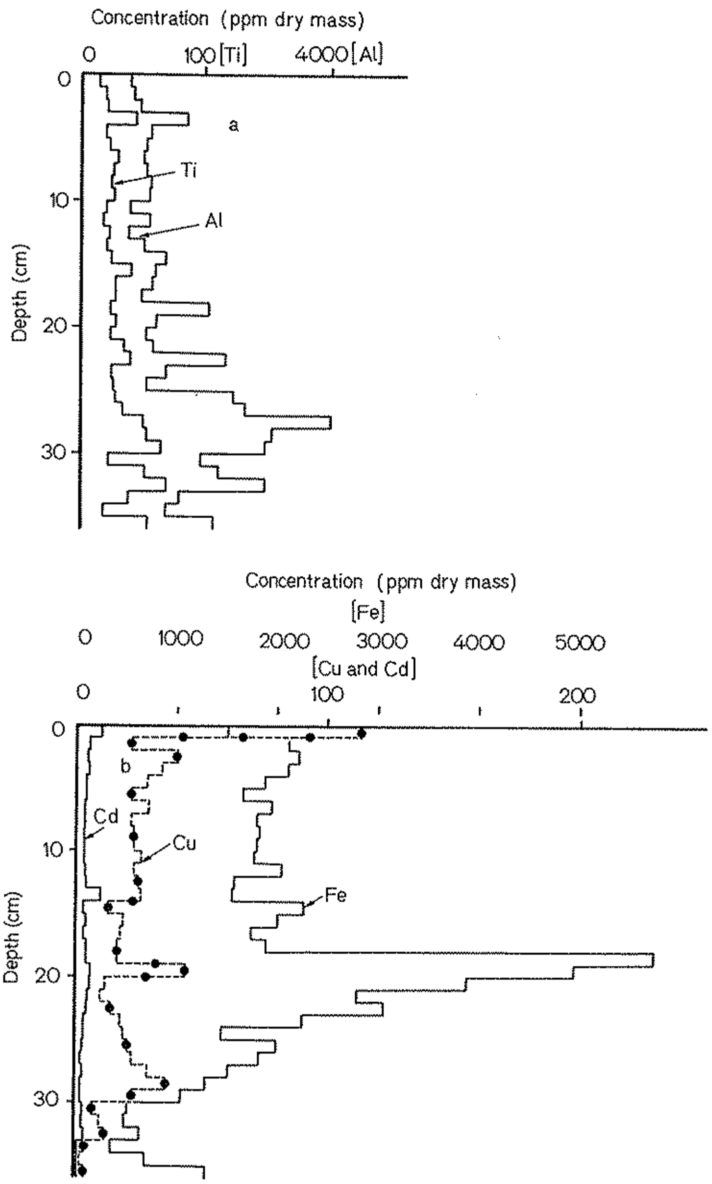


Fig. 6 a–d. Bulk density and concentrations of Ti, Al, Cd, Cu, Fe, Mg, Pb, and Zn in a 21 cm diameter core from a *S. magellanicum* lawn on Burnt Hill, Moor House National Nature Reserve

Bulk density increases fairly steadily with depth from 0.01 g cm^{-3} at the surface to 0.06 g cm^{-3} at 28 cm. Below this it increases sharply to 0.2 g cm^{-3}—a value as high as any recorded for organic peat. Above 28 cm *Sphagnum* leaves could be seen, but below 28 cm they could not be found even with a microscope.

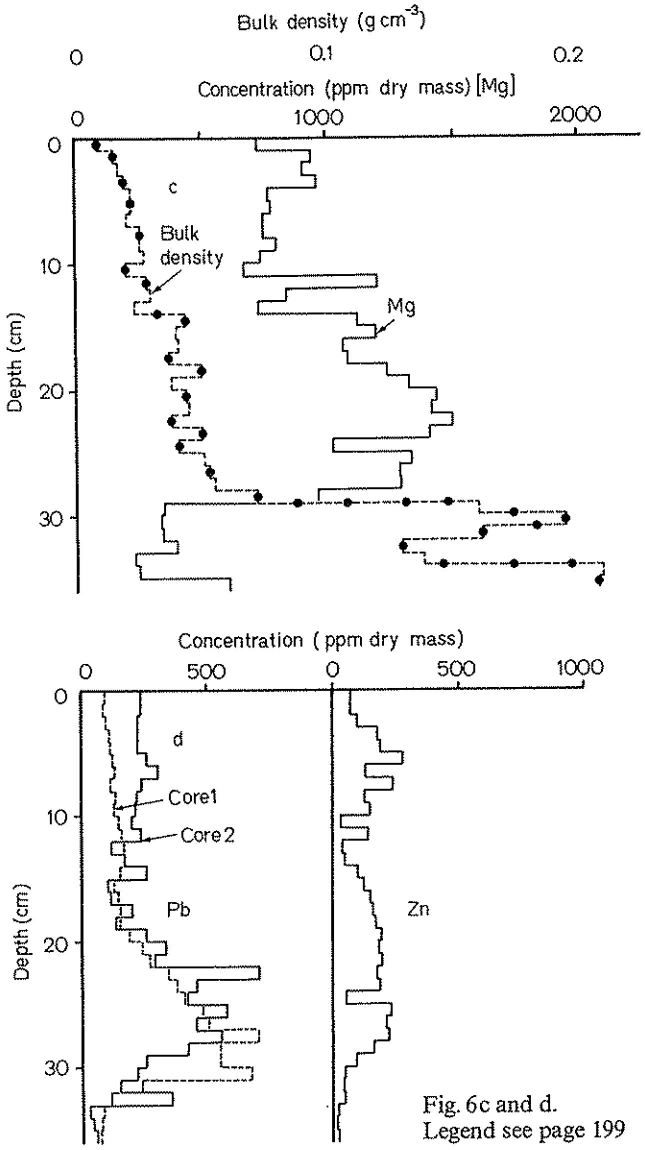


Fig. 6c and d.
Legend see page 199

There was a marked visible change in colour (from light brown to dark brown) and in texture (from rough to smooth) at the same level. A similar discontinuity is seen in the concentrations of Al, Ti, Mg, Pb (and perhaps Zn).

If conditions had been constant during the formation of the whole core it would be surprising to find the sharp change in bulk density and loss of structure at 28 cm, though it is possible that this is a critical point at which structure collapses. The associated decrease of concentration of so many elements of such diverse origins makes such an explanation unlikely. More probably there was at this time a marked change in environment, possibly connected with pool drainage. The model assumes that the parameter values do not change so it seems best to apply the model to the top 28 cm only of this core.

Age Profile. A continuous age profile may be established from the cumulative total amounts of Al, Ti, and Mg calibrated by the date of a single horizon established from a peak in Cs-137 concentration corresponding to the peak in fallout in 1963. Details are given in Section 9.5.3.

The Al, Ti, and Mg concentrations may also be used to check the aerobic decay rate α_1. If the deposition rate has been constant then from Equation (3):

$$\alpha_1 \simeq \log(C_o/C_t)/t \tag{9}$$

where C_o is the concentration of metal at the surface, and C_t the concentration at a depth of age t, the concentrations being mass of metal per unit mass of dry peat. Using the 0–2 cm and 11–13 cm depth, with $t = 9$ yr, the third estimate in Table 2, $\alpha_1 = 0.062$, is obtained.

Direct Method Comparison of Observed and Predicted Profiles. The bulk density and age profiles predicted by the model, using the direct method, may now be compared with the observed bulk density, the age at 12 cm, and less reliably with the Al/Ti/Mg age profile. Figure 7 shows observed values and ones predicted using the "amorphous peat" [Eq. (5a)] model. The $P = 0.05$ error bounds on the observed profiles are those for the least squares polynomial with the same number of parameters as the model.

The predicted bulk density profile is in fair agreement with the observed one to about 14 cm depth, but below that level the predicted profile is systematically below the observed one. The predicted age is systematically too great near the surface and too little lower down.

The age profile comparison is important because for this particular observed bulk density profile the model contains a high degree of redundancy. A linear regression (with intercept) accounts for 0.85 of the variance, whilst the sixth degree polynomial accounts for only 0.05 more. The age profile provides a comparison independent of the bulk density.

The effect of stimuli (changing the parameter values) on the "amorphous peat" model is shown in Table 3 and effects on the "fibrous peat" model, using Equation (5b) in Table 4. The ratio of predicted/observed bulk density is shown at four depths. These were selected so that the observed bulk density was close to the trend line at that depth. The shallowest (4 cm) shows whether the predictions are seriously in error near the start. The 12 cm depth is one for which the age is best established. If the bog did start growth anew on a bare peat surface at 28 cm it would be reasonable to suppose that productivity, at least, varied in the early stage of re-establishment. The 22 cm depth is some way above such a point. The 28 cm level was chosen for reasons already described.

It is apparent from Tables 3 and 4 that changing the decay or creep parameters has relatively little effect on the bulk density predictions. The creep parameter has little effect on predicted age either, but the aerobic decay parameter α_1 has a marked effect on age. Varying the compression parameter k or water table W has a bigger effect, but the model is most sensitive to variations in productivity p, and the rate of growth in length L.

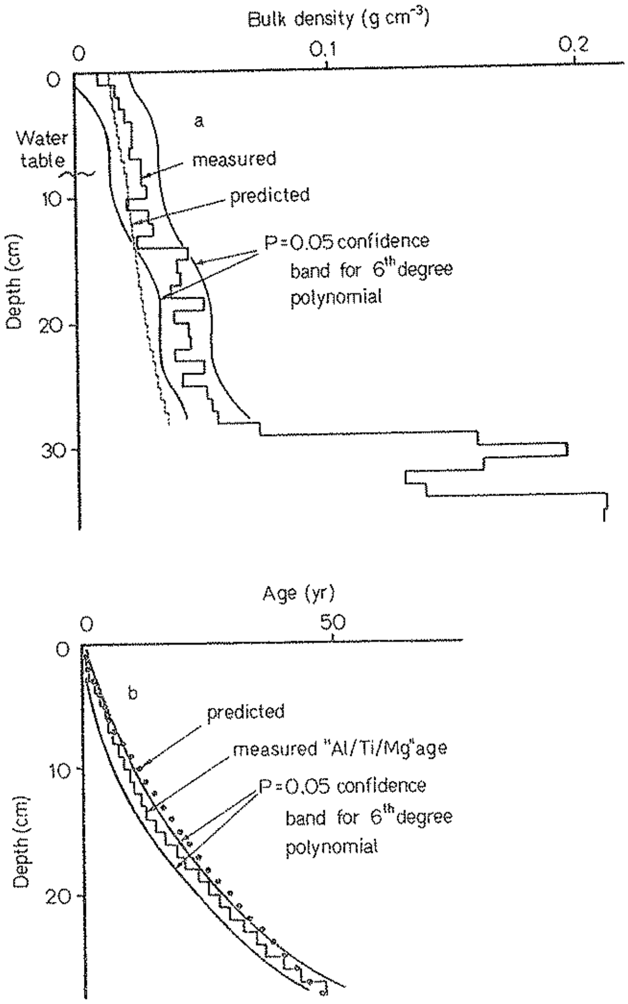


Fig. 7 a and b. Measured and predicted bulk density and age profiles for the same core as Figure 6 using measured parameter values and the "direct" method—see text. The P = 0.05 confidence bands for a 6th degree polynomial fitted to the observed values is also shown. For the "Al/Ti/Mg age" the polynomial was fitted to the concentration for each slice and the band shown was obtained from the cumulative error variance

Very approximately one may say that the ratio p/L determines the general position of the predicted profile, whilst the slope and curvature are determined by k and to a lesser extent c and indirectly, through effects on stress and creep time, by α. Water level has an indirect effect by altering the time for aerobic decay and by affecting the mass of peat exerting its full weight in air.

None of these tests produces a good fit to both the observed bulk density profile and to the age profile, although it seems that the age profile might be fitted by varying α_1 with relatively little effect on the bulk density profile.

Table 3. Effect of varying basic parameter set of Table 2 in “amorphous peat” model

	Bulk density: predicted/observed				Age (yr)	
Depth (cm)	4	12	22	28	12	22
Observed bulk density (g cm^{-3})	(0.020)	(0.029)	(0.045)	(0.055)		
set=	1.0	1.0	1.0	1.0	9	30
Basic set	0.78	0.75	0.64	0.64	13.9	32.5
0.5 p	0.36	0.29	0.21	0.18	11.5	24.0
1.5 p	1.28	1.59	2.10	4.56	18.0	53.1
0.5 L	0.53	3.77	np	np	174.0	np
1.5 L	0.49	0.42	0.31	0.28	7.4	15.5
0.5 α_1	0.78	0.75	0.64	0.64	12.0	26.9
1.5 α_1	0.78	0.75	0.64	0.64	16.9	41.3
0.5 α_2	0.78	0.75	0.64	0.64	nr	32.5
1.5 α_2	0.78	0.75	0.64	0.64	nr	32.5
0.5 k	0.72	0.58	0.41	0.36	11.5	24.0
1.5 k	0.85	1.06	1.40	3.04	18.0	53.1
0.5 c	0.78	0.74	0.62	0.62	13.8	32.0
1.5 c	0.78	0.76	0.65	0.66	14.1	33.0
W= 4 cm	0.78	0.65	0.53	0.41	11.2	23.6
W=12 cm	0.78	0.87	0.79	0.87	15.9	50.2
W=16 cm	0.78	0.87	1.11	1.35	nr	134.5

np: no prediction; nr: not relevant.

Table 4. Effect of varying basic parameters of Table 2 in “fibrous peat” model

	Bulk density: predicted/observed				Age (yr)	
Depth (cm)	4	12	22	28	12	22
Observed bulk density (g cm^{-3})	(0.020)	(0.029)	(0.045)	(0.055)		
set=	1.0	1.0	1.0	1.0	9	30
Basic set	0.87	1.19	2.07	np	19.6	65.6
0.5 p	0.38	0.33	0.26	0.24	12.8	28.3
1.5 p	1.53	8.29	np	np	59.1	np
0.5 L	2.49	np	np	np	np	np
1.5 L	0.53	0.52	0.46	0.49	8.6	20.0
0.5 α_1	0.87	1.19	2.07	np	16.1	51.3
1.5 α_1	0.87	1.19	2.08	np	25.6	91.8
0.5 α_2	0.87	1.19	2.07	np	nr	65.5
1.5 α_2	0.87	1.19	2.07	np	nr	65.6
0.5 k	0.76	0.67	0.52	0.49	12.8	28.3
1.5 k	1.02	5.50	np	np	58.9	np
0.5 c	0.87	1.19	2.06	np	19.6	65.4
1.5 c	0.87	1.19	2.09	np	19.6	65.7
W= 4 cm	0.87	0.84	0.97	1.51	13.3	32.7
W=12 cm	0.87	0.84	20.2	np	30.7	700

np: no prediction; nr: not relevant.

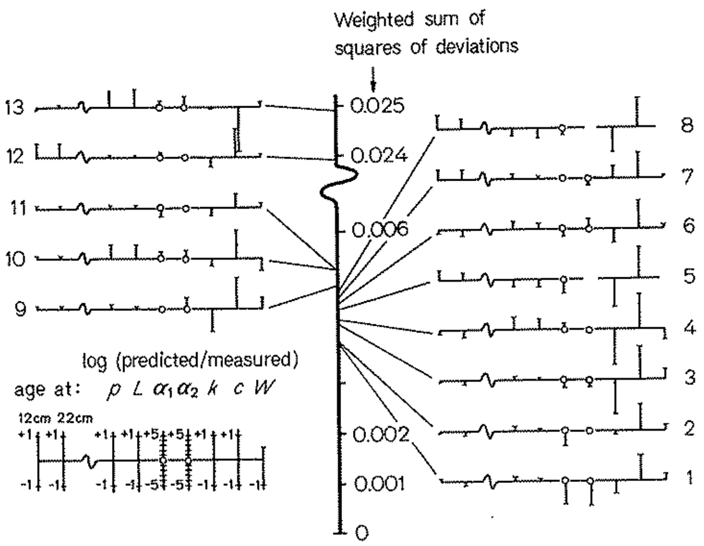


Fig. 8. Agreement between observed and predicted values of parameters, and (independently) of age at two depths using the "indirect" method (see text) on the same core as in Figure 6. Thirteen solutions of varied success are shown. *Vertical bars* show predicted/observed values on a logarithmic scale. The scale of the bars for the decay parameters α_1 and α_2 is one-fifth that of the others. The shorter the bars, the better the agreement. In solutions 5 and 8 the predicted water table, W, was at the bottom of the core, so the parameter α_2 has no value

Indirect Method Comparison. It is now worth examining the results of the indirect tests, in which only the bulk density profile was supplied, and best estimates of the parameters were made.

Because this is an expensive procedure, it was not considered worth-while testing both "amorphous" and "fibrous" peat models. The "amorphous peat" model was chosen because it was the more stable, produced results with the direct tests which were closer to the observed ones, because in any extension to other peats and depths the "amorphous" peat predominates, and because the rate of creep of the fresh *Sphagnum* peat of this core (Fig. 2b) seems in practice to be almost independent of applied stress in the range of interest.

The first indirect tests were made with all seven parameters (p, L, α_1, α_2, k, c, W) free to vary. A lower limit of 0.0 (or 0.1 cm for W) was set, to avoid wasting time with unreal (negative) values. An upper limit of 200 (or, in the case of W, the depth of the core) was set for the same reason. Start points varied by a factor of 100 on either side of the measured parameter values. These constraints are very light ones. Thirteen runs were made, and each located a different minimum (Fig. 8). For each solution the ratio of "best fit"/measured value for each parameter is shown, and the same ratio for age at 12 cm and 22 cm depth (using the "Al/Ti/Mg age" already described). A log scale is used, that for the decay parameters being five times smaller than for the others. The shorter the bars, the better the agreement between measured and "best estimates". It is worth emphasising that

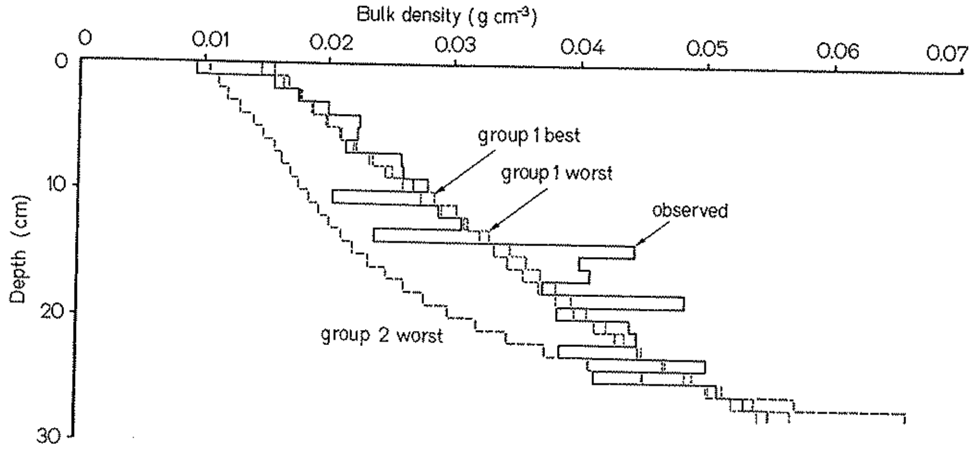


Fig. 9. Observed bulk density and that predicted for solutions 1 ("group 1 best"), 11 ("group 1 worst"), and 13 ("group 2 worst") of Figure 8

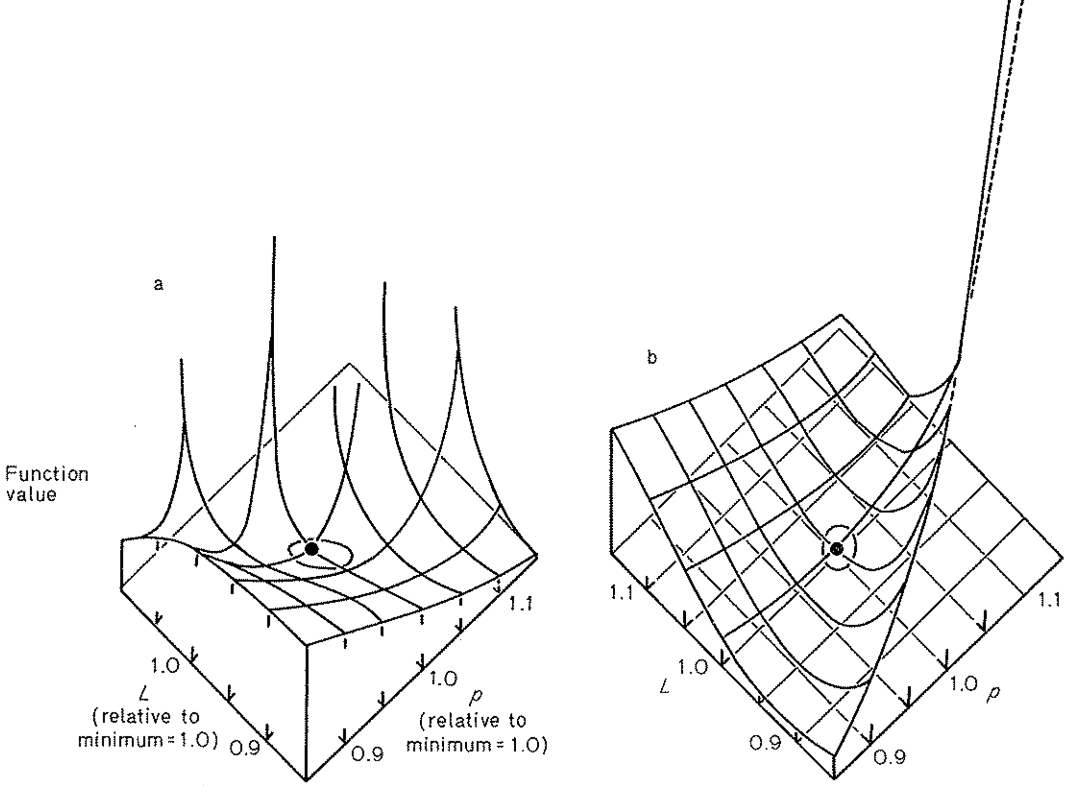
the "best estimates" are based solely on the bulk density profiles and, with the light constraints, differences from the measured values (except for W) of tens of orders of magnitude might occur.

The solutions are in two groups. The sum of weighted squares of deviations for the lower 11 are within a small range; the largest is 1.38 times the smallest. The other two are six times the smallest. The bulk density profiles for the least and greatest in the first group, and for the worst of all, are shown in Figure 9. It seems unrealistic to claim that any solution within the first group is demonstrably preferable to any other, but the second group are a distinctly worse fit to the bulk density profile. One may also conclude that in view of the general variability of the observed profile, and the necessarily smooth nature of any predicted profile, it is unlikely that there is any solution which is a much better fit than any yet found: that the hypersurface does not have any much lower points on it.

The hypersurfaces around the minima—Figure 10a shows an example for the two parameters which were always most precisely estimated—is some way from parabolic, so the correlation matrices must be interpreted with caution. There is much intercorrelation: in six solutions five values (out of the 21 possible) are greater than 0.6 or less than -0.6. The correlation illustrated in Figure 10a between productivity and length growth rate is 0.98. Since a linear least squares regression provides a good fit this is not surprising. Although other solutions show similar numbers of high correlations, there is no obvious pattern: it is not always the same pairs of parameters which show a high correlation.

Taken together these results suggest that the surface is equivalent in two dimensions to an elongated trough with an uneven floor of shallow depressions with no clear pattern. The trough seems to be broader for some parameters (α_1, α_2) than for others, since the range of solutions is much greater for them, and the precision of the estimates of α_1 and α_2 in any one solution is low.

Fig. 10 a and b. Two-dimensional surfaces of minimisation function in cases showing the highest correlation between parameters. The parameter units are relative to a value of 1.0 at the minimum in all cases. A contour is sketched close to the minimum. In (a) (solution 6 of Fig. 8), where only the deviations from bulk density were minimised, the contour is much elongated and off centre. In (b) where both bulk density and age deviations were minimised, the contour is less elongated and less off centre. Ideally the contour would be circular and centred on the minimum



In view of the enormous possible range of the parameters, however, the convergence of p, L, k, c and to a lesser extent W on a relatively narrow range may perhaps indicate that the model is moderately realistic.

That the general region of the hollow coincides with the measured position (in many cases) is additional and independent support. The decay parameters however are not centred on a well-defined region. The major anomaly is in the estimates of the creep parameter, which are consistently an order of magnitude greater than the measured values. This may indicate that the rate of creep in partly humified peat is markedly different from that measured in unhumified peat.

With 21 correlations, and only 28 observations, it seemed desirable to reduce the number of parameters. Water table level seemed the obvious one to choose. In the real system it is as tightly constrained as any; there is a tendency for solutions

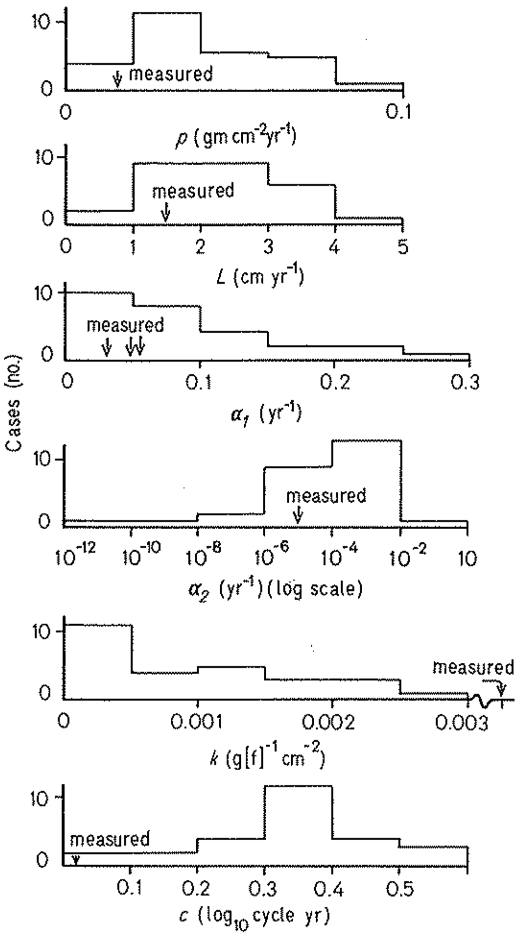


Fig. 11. Histogram showing distribution of 28 sets of parameter estimates, using the "indirect" method (see text) and the same core as shown in Figure 6, but fixing water table, *W*, at 8 cm. *Arrow* = measured values

to be found in the impossible situations of water table above the surface or below the bottom of the peat, which then inactivates one or other of the decay parameters; and it is a parameter whose value can be relatively easily measured.

A second set of tests was therefore made on the same data, but with the water table fixed at 8 cm; see Figure 11. The weighted sum of squares of deviations ranged from 0.0042 to 0.0053 (compared with 0.0038 to 0.0052 for the first group of solutions). The results are in general similar to the seven parameter tests, except that α_1 now shows a much reduced range, as might be expected. The degree of parameter correlation was also reduced; only two were outside the range -0.6 to $+0.6$. The agreement with measured parameter values was not very good, however, when the effectiveness of a linear regression fit is recalled.

Table 5. Best estimates of parameters minimising deviations from both age and bulk density

Parameter	Solution			Measured value
	I	II	III	
p	0.029	0.032	0.032	0.016
L	1.53	1.78	1.80	1.5
α_1	0.020	0.038	0.039	0.032, 0.050, 0.062
α_2	0.4×10^{-7}	0.0065	0.0054	10^{-5}
k	0.0015	0.0022	0.0022	0.005
c	0.033	0.0001	0.0023	0.018
W	18.7	12.0	12.1	8
p'	0.018	0.021	0.021	
L'	0.44	0.64	0.64	

The extent of intercorrelation of parameters may also be reduced if deviations from both bulk density and the complete Al/Ti/Mg age profile are minimised simultaneously.

A third set of tests was, therefore, made. The random error in the Al/Ti/Mg measurements (judged from the residual sum of squares of deviations from a sixth degree polynomial) was about the same (0.10 of the total) as it was in the bulk density measurements. There was no need therefore to give different overall weights to bulk density and age, though the individual measurements of age were given weights in the same way as were the bulk density measurements. The quantity to be minimised was the sum of the geometric mean of the squares of the weighted deviations at each level. The hypersurfaces of the minima (for example Fig. 10b) were much closer to parabolic than in the first tests. Intercorrelation of parameters was much reduced too; in the best case the most significant correlation (shown in Fig. 10b) was only −0.42 and in no others reached the 0.25 level. The spread of the weighted sum of squares of deviations was much greater (for the same starting points as in the first group in Fig. 8), from 0.71 to 5.13. The three best results, all nearly equally good, are shown in Table 5. Again, bearing in mind that the parameters might vary over tens of orders of magnitude, the agreement of estimated and measured parameters is fair, except for α_2 and c which are in any case rather imprecisely estimated.

To provide an equally good polynomial fit to both bulk density and age profiles also needs five parameters (two for bulk density and three for age).

In conclusion the model predicts profiles or parameters which are in moderate agreement with the measured ones. Whether the differences arise from deficiencies in either or both the model and measurements is unclear.

Investigations on Other Sites. The tests described have used data from a single site. The results might be due to chance properties of this data set, so it seemed important to investigate other sets too.

Three groups of cores have been examined. First, three cores taken from Moor House using the same methods as already described, and for which Cs-137 pro-

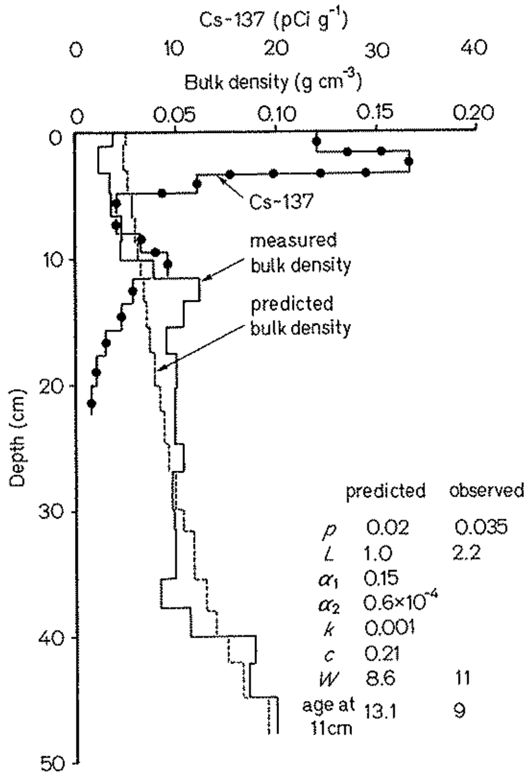


Fig. 12. Observed and predicted bulk density, and Cs-137 concentration for a core (MH2) from a *S. papillosum* lawn at Moor House National Nature Reserve. The "indirect" method (see text) was used, and this is the best of three solutions. The predicted, and three measured, parameter values are shown. The observed age is obtained by taking the Cs-137 peak as representing 1963

files were made. Second, cores made to much greater depths using a Russian pattern (West, 1968) borer (Chap. 8). For one of these cores two C-14 ages are available. Third, cores from other areas; the only one reported here is from Abisko (N Sweden), and a C-14 date is available for it too.

Short Cores from Moor House. MH2 came from a *Sphagnum papillosum* lawn on Burnt Hill, about 200 m from the cores already examined in detail (grid ref NY753329). MH5 came from a *S. papillosum* lawn in a small bog about 250 m from Green Burn (grid ref NY774322). It should be emphasised that the core site is not Forrest's site, described in Smith (1973). MH6 came from a *S. magellanicum* hummock, about 3 m diameter, 5 m from MH5.

The model was fitted using the indirect method (minimising a weighted sum of squares of deviations of the observed bulk density profile from that predicted). The results from the best of three runs for each core are shown in Figures 12, 13, and 14. Measurements of *p*, *L*, *W* and the Cs-137 1963 peak were available. The agreement with predicted values is again moderately good, with the same reservations as for the first core.

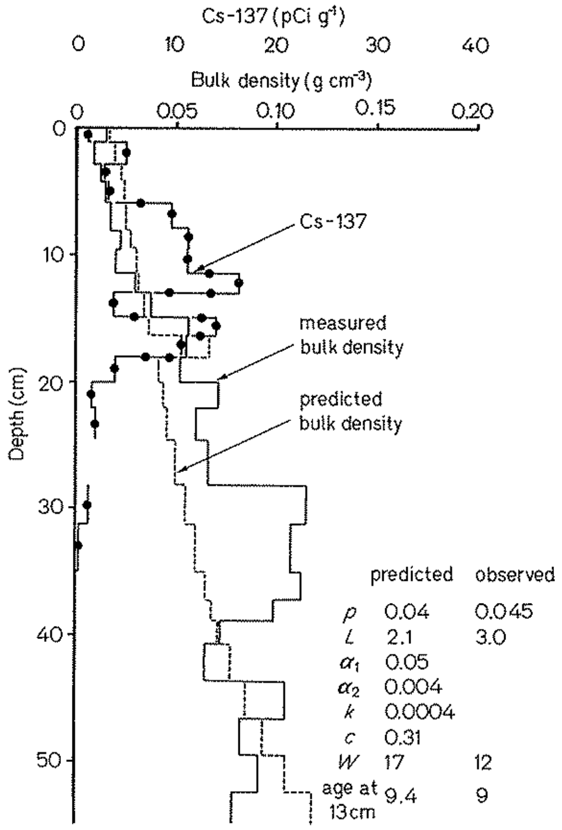


Fig. 13. Similar to Figure 12, for a core (MH5) from *S. papillosum* lawn

Long Cores from Moor House. The "steady state" assumptions underlying the model reported here cannot be expected to hold for the ages and depths of the long cores from Moor House blanket bog peat (Chap. 8). Nor is it to be expected that the functional relationships can be extrapolated far. Nevertheless, it seemed of interest to see how far the model could be stretched. The indirect method of fitting was therefore applied to Jones and Gore's Valley Bog data (Chap. 8, Table 8.13). The best of three results is shown in Table 6. The estimates of k and W seem to be far from the likely real values, and the predicted ages are low by a factor of 3. For these colossal extrapolations however it is perhaps surprising that there is even order of magnitude agreement.

Core from Abisko. Finally the model was fitted to bulk density data, collected by Prof. Mats Sonesson, from Stordalen, Abisko, N Sweden. The same method as before was used. The best of four results is shown in Table 7. This core is interesting because the bulk densities are much greater than others used so far, and the peat was formed in very different climatic conditions. The measured parameter values are in better agreement with the estimates than were those from long cores at Moor House.

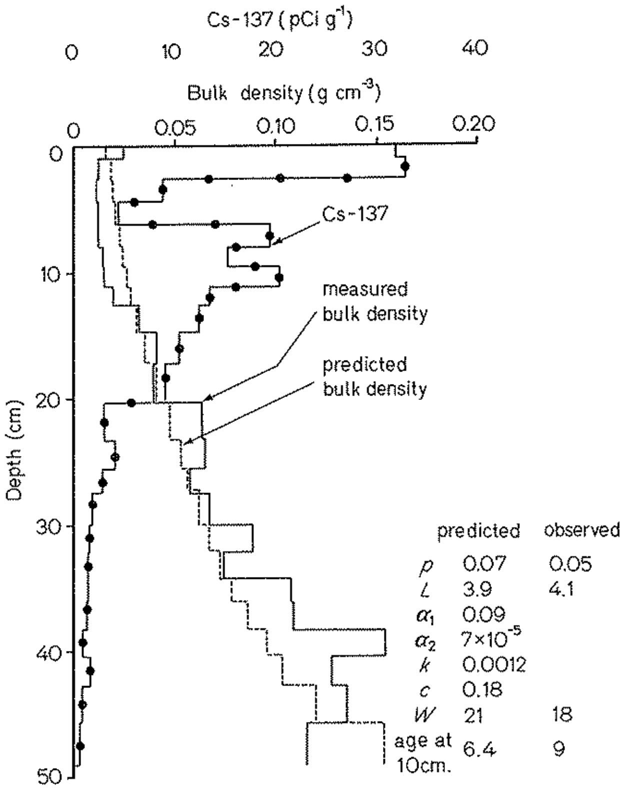


Fig. 14. Similar to Figure 12, for a core (MH6) from a *S. magellanicum* hummock

9.3 General Discussion

This model, although crude, produced predictions which were in moderately good agreement with measurements. The sensitivity tests (Tables 3 and 4) indicate that the parameters which most affect peat bulk density are productivity and length growth rate. Next in importance are variations in water table and in the compression parameter. Least important are the creep and decay parameters. The importance of water table variation can be compared directly with the finding by Jones and Gore (Chap. 8) that in their model the slope of the regression of decay rate on depth has a large effect on the bulk density profile. It is not so much the rate of aerobic decay but the length of time for which it operates which matters. This is not a new conclusion: the model reflects old beliefs. This point is emphasised when one examines the apparent average rate of peat accumulation (Table 8). There are two types of estimate of peat growth included. The C-14 ones are simply average rates. The first three sites in Table 8 were chosen from many available (Walker, 1970, gives a longer list) for relevance to the material in this paper, or in the case of Tregaron because a wide range of rates is shown at one site. The other two estimates are of the annual increment to the anaerobic zone.

Table 6. Best estimates of parameters for Valley Bog, Moor House

Parameter	Estimate	Measured
p	0.034	
L	1.4	
α_1	0.028	
α_2	1.5×10^{-5}	
k	0.00013	
c	0.202	
W	27	
Age at 160 cm (yr)	604	2,200[a] ± 50
Age at 310 cm (yr)	1,500	4,700[a] ± 57
p'	0.011	
L'	0.97	

[a] Turner et al. (1972).

Table 7. Best estimates of parameters for Abisko, N Sweden

Parameter	Estimate	Measured
p	0.033	
L	0.32	0.4
α_1	0.004	about 0.03
α_2	10^{-5}	
k	0.004	
c	0.23	
W	32	30
Age at 36 cm	637	1,110
Age at 72 cm	3,770	5,240
p'	0.0048	
L'	0.019	

Table 8. Some estimates of peat growth rate

Place	Method	Growth rate (cm yr^{-1})	Growth rate (g cm^{-2} yr^{-1})	Source
Abisko (Sweden)	C-14	0.01		Sonesson (unpubl.)
Moor House Valley Bog (UK)	C-14	0.057 to 0.098		Turner et al. (1972)
Tregaron (Wales)	C-14	0.01 to 0.3		Turner (1964)
Glenamoy (Ireland)	p, α_1 and W model	0.04[a]	0.0032[a]	Moore (1972)
Moor House MHI data (UK)	7 parameter model	0.4[a]	0.018[a]	This paper

[a] These are the predicted input to the anaerobic zone.

Since these make no allowance for later decay and compaction, one would expect them to be greater than the C-14 averages. The estimates from the model used on the core MH1 are, however, well above most of the C-14 estimates. Those for Glenamoy are close to the C-14 estimates. The measured aerobic decay rate is slightly greater at Glenamoy though productivity is similar at both sites. The Glenamoy model, however, allows aerobic decay to operate to a depth of 25 cm. This points to one of many weaknesses in this *Sphagnum* peat model: it has been assumed that the water table is at a fixed position, and that this is coincident with the change from aerobic to anaerobic decay. In fact of course the transition is probably some way below the water table, and the water table fluctuates during the year (Clymo, 1965; Goode, 1970; Forrest and Smith, 1975) being lower in summer. Decay is faster during the summer in field conditions (Clymo, 1965) and at higher temperatures and higher water content in laboratory experiments (Rosswall and Berg, 1973). In consequence the mean water level is probably well above the notional aerobic/anaerobic transition.

The indirect method of model testing, which seems attractive at first sight, proves less helpful than hoped because the shape of the bulk density profile is relatively simple. Using the age profile as well gives a marked improvement and it would be valuable in any development to introduce a third variable if possible. This is desirable because it is likely that all realistic peat growth models based on steady state hypotheses will produce predictions which are monotonic functions of depth. The present model does not produce monotonic predictions—when the creep term begins to exceed the others the bulk density becomes positive infinite then negative—but this is simply due to extrapolating beyond the limits of physical reality. It is indeed surprising that predictions on the long and older cores are as close to observation as they are.

Walker (1970), having examined the apparent average rate of peat accumulation of bog peats at various depths below the surface, concluded tentatively that "there is no correlation between sample depth and apparent accumulation rate confirming that, beyond the earliest stages of deposition, progressive compaction does not normally occur". Kaye and Barghoorn (1964) expect autocompaction in *Sphagnum* peat to be negligible, and refer to a C-14 dated core indicating a steady rate of accumulation of about 1 mm yr^{-1}. Lee and Tallis (1973) find that the rate of peat accumulation at a site in the south Pennine hills has been roughly constant for three hundred years to the present time.

By contrast the age profile for the *Sphagnum* peats (reported here) indicates that compaction *has* occurred in the top 0.5 m at least, accompanied by a five-fold increase in bulk density. The time scale of this change is relatively so short however that it is still consistent with the conclusions of Walker and of Kaye and Barghoorn. The peat examined by Lee and Tallis has a present day cover of *Eriophorum*, and the model parameters for such a system are probably very different from those reported here.

The great importance of events in the top 50 cm of these ecosystems as determinants of peat accumulation rates is indicated.

There are some unresolved difficulties however. A rate of accumulation constant over thousands of years is rather surprising when one remembers the concomitant changes in climate. Furthermore, if the hypotheses of Granlund (1932) and Wickman (1951) concerning the maximum height for bog growth are correct, one might expect a generally decreasing rate of peat accumulation with time. In the model presented here it is possible for the age to vary independently of the bulk density and the sensitivity of age to variation in parameter values is not always the same as that of bulk density. In particular the age is more sensitive to variation in the aerobic decay rate, α_1, than is the bulk density. The consequence is that a change in peat accumulation rate might leave no trace in the bulk density profile. The reciprocal effect—a change of bulk density such as may be found at a recurrence horizon—with less marked change in accumulation rate is not predicted.

The bulk density profiles, to depths of 3 m and more, reported by Jones and Gore (Chap. 8) do seem to show a gradual increase of bulk density with depth. If the model were applicable to such cases one would expect a curvilinear relationship between age and depth.

Compaction, selective decay of less dense materials, and progressive changes in climate, plant cover and bog surface microenvironment may all contribute to the age and bulk density profiles. The model described here has allowed only the first of these. It is hardly surprising therefore that the whole range of phenomena concerned with peat growth cannot yet be reconciled.

9.4 Conclusion

A peat bog gains dry matter by growth of plants at the surface, and loses it by decay. Rate of growth in depth is the net result of gain by plant growth, and loss by decay, primary compression and secondary consolidation (creep). The evidence for functional relations is presented, and a seven-parameter model constructed.

The data used for checking the model are parameter measurements, bulk density profiles, and an age profile, based on measurements of Cs-137 and on Al, Ti, and Mg concentrations.

The model is used in two ways:

1. Given measurements of the parameters, age and bulk density profiles are predicted.

2. Given a bulk density profile, best estimates are made of the seven parameters and the age profile. Alternatively both bulk density and age profiles are supplied, and the seven parameter values estimated.

Productivity, the rate of growth in length of plants, aerobic decay rate and water table position seem to be the most important parameters in this model.

The importance of the surface layers in determining peat accumulation rates is emphasised.

Acknowledgements. I record my thanks to Mr. R.S.Cambray and Mr. M.C.French for making the estimates of Cs-137 and lead; to Mr. E.J.F.Reddaway for the estimates of carbon dioxide and methane flux and for some of the growth estimates; to Mrs. P.Ratnesar for expert and unfailingly careful assistance with most of the other experimental work; to Dr. P.Osmon for guidance with the radioactive counting; to Prof. Mats Sonesson for permission to use an unpublished peat density profile; to Prof. J.Essam, Dr. D.Knowles and Dr. C.Place for help with mathematical problems; to Dr. P.Cawse, Dr. K.E.Clymo, and Mrs. B.Thake for commenting on the typescript, and to other colleagues for occasional help in many ways.

I am grateful to the Natural Environment Research Council for financial support for part of this work.

9.5 Appendix

9.5.1 Model Construction

The justification for functional relations used is given in Section 9.2.1. Here they are assumed.

Dry Matter

$$\frac{dx}{dt}=p-\alpha x \tag{A1}$$

where p = rate of addition of dry matter per unit area
x = total accumulated dry matter per unit area
α = decay parameter
t = time since growth began.

The solution of Equation (A1) is

$$x=\frac{p}{\alpha}(1-e^{-\alpha t}). \tag{A2}$$

It is implicit in Equation (A2) that as t increases, x approaches p/α which may be called the steady state mass.

Depth. Consider a finite slice of peat which is now, at n time units, of thickness Z; and which was formed j time units from the start. When first formed it was of thickness Z_1. Then if T represents time in discrete units from 1 to n,

$$Z_j=Z_1 f(T), \quad f(1)=1. \tag{A3}$$

This equation describes compaction with time for a single slice. The peat bog is assumed to have a new slice added each time unit, so the depth of peat, l, is given by:

$$l=Z_1\sum_{j=1}^{n} f(T_n-T_j). \tag{A4}$$

Now consider the process as a continuous one. Analogous to Equation (A 4) we may write:

$$l=L\int_0^t f(t-t')\,dt' \tag{A 5}$$

where Z_1, the depth added in unit time, is replaced by L, the instantaneous rate of addition, and T_n and T_j are replaced by t and t'.

Then:

$$\begin{aligned}\frac{dl}{dt}&=L+L\int_0^t f'(t-t')dt'\\&=L-L[f(t-t')]_0^t\\&=Lf(t).\end{aligned} \tag{A 6}$$

This simple but unobvious result allows one to write the rate of growth of the whole peat column given simply a knowledge of the behaviour of one part, provided that this is a function of time alone. This is obviously so for creep [Equ. (5)].

For compression, the stress in Equation (4) is due to the weight of peat (per unit area) above the slice. Where the peat is under water, the stress ε is:

$$\varepsilon=nxg(1-\rho_m/\rho_s) \tag{A 7}$$

where g is the weight per unit mass ("acceleration due to gravity") at this point on the Earth's surface, ρ_m is the density of the medium and ρ_s is the density of the peat dry matter; 1.6 g cm^{-3} for *Sphagnum* (Clymo, 1970). The factor n (explained below) is 1. The quantity x, a function of time, may be found from Equation (A 2).

Where the peat is in air above the water table, the effective weight is due not only to the dry matter, but also to associated water in capillary spaces. Field measurements show that the water content of *Sphagnum* varies from about 10 times the dry mass at the surface to about 40 times just above the water table. An empirical function could be used, but implementing the model is then more awkward and much more time consuming, so for simplicity the approximation that the water content is on average 20 times the dry mass has been used. For computations using Equation (A 7) above the water table ρ_m is $\simeq 0.0$, and $n = 20$.

In some circumstances the stress of snow and ice might be very important. Certainly *Calluna vulgaris* at Moor House can be crushed in this way. West (1968) gives the density of old snow as about 0.2 g cm^{-3} so a 100 cm deep drift might produce a stress of 20 g (f) cm^{-2}, which is roughly equivalent to the same depth of wet *Sphagnum* in air. If the surface peat layer is frozen the stress acts at an unknown depth in the peat. The stress is temporary however, and for want of knowledge of its effects has not been included in the model. Compression can thus be written as a function of time.

For decay, the analogue of (A 3) for slice thickness as a function of time may be written.

Assuming that all four processes operate independently, and making use of Equation (A 6) gives:

$$\frac{dl}{dt} = L \exp\left\{-\alpha t - kng(1-\rho_m/\rho_s)\frac{p}{\alpha}(1-e^{-\alpha t})\right\}(1-c\log[m+t]) \qquad \text{(A 8)}$$

for "amorphous peat". For "fibrous peat" the term log $[m+t]$ is multiplied by the stress. No explicit solution for Equation (A8) is known, but it may be solved numerically.

Aerobic and anaerobic layers are incorporated by redefining a local t at zero as the water table is passed at depth W. At this time the mass of aerobic peat has reached a steady value, and so has the contribution to compressive stress from this source. A new value of p, which now represents the input to the anaerobic zone, is calculated from Equation (A 1) whilst ρ_m becomes 1.0 g cm^{-3}. The parameter α, which was α_1 relating to aerobic conditions, is replaced by α_2 relating to anaerobic conditions. The calculations are continued, adding an extra term in Equation (A 8) for αt at the water table, and another for the stress due to material in the aerobic zone. Time for the creep term refers to the original time.

Implementing the model: A FORTRAN subroutine takes values of the parameters $p, L, \alpha_1, \alpha_2, k, c$, and W and a series of depths defining slices of peat parallel to the surface. Between the two values of l defining each slice and the value of t for the upper surface, Equation (A 8) is integrated to give t for the lower surface. A simple Gaussian quadrature method could be used, but in practice to allow possible addition of terms involving l, a fourth-order Runge-Kutta technique (Merson, 1957) is used, involving a fifth evaluation of Equation (A 8) from which some estimate of error is derived. The Runge-Kutta method was chosen because the integration needs restarting for each depth, must finish at a predetermined time or depth, and need not be of accuracy better than about 1 in 10^4. These requirements are not so well satisfied by predictor-corrector, nor by polynomial extrapolation methods. In critical cases a check is made by repeating the calculation with reducing step sizes. The step size is normally adjusted to minimise computing time. From this value for t, and using Equation (A2) x is calculated. The bulk density of the peat in the slice may then be calculated.

This process is repeated for each slice. The main output from the subroutine is a series of times for slice boundaries, and bulk densities of slices. The use made of these is described in Section 9.2.2.

9.5.2 Methods

9.5.2.1 *Minimisation Technique*

The CERN program MINUITS (library D 506, D 516) was employed, using the simplex option (Nelder and Mead, 1965) to locate minima, and Davidon's (1968) method to refine the estimate and to estimate parameter correlations. Davidon's method assumes that the surface is quadratic about the minimum. In a few cases values about the minimum were calculated to investigate this point.

The quantity minimised was usually a weighted sum of squares of differences between observed and calculated bulk density. The weighting was made proportional to layer thickness, and inversely proportional to the variance about the straight line of best fit of the nearest five observed densities and depths. This weighting was used to equalise the deviations as far as possible in data sets with different characteristics.

9.5.2.2 *Bulk Density Profiles*

Sites were selected where *Sphagnum* formed nearly pure carpets. Cylindrical cores 21 cm diameter down to 75 cm were taken using a cylindrical cutter with 3 cm long teeth with sharpened edges. The cores were removed either by forcing a spade below the cylinder and hauling out at an angle of 45°, or using a network of nylon cords drawn across the bottom of the core. The first method is simpler, and was used if the peat was coherent or fibrous. The second method worked better on semi-fluid humified peats.

Cores were transferred to plastic tubes with wooden bases to minimise loss of water (and consequent compaction) during transport.

It was possible to check the length of the cores at each stage. Any cores showing a change of more than 1 cm were rejected; these were mostly from very wet sites with *Sphagnum cuspidatum*.

The cores were frozen solid, then cut into slices with a sharp carpenter's saw. Distances were all measured from the core base, so errors in slice thickness should be neither cumulative nor proportional to thickness.

The slices were allowed to thaw, separated into small pieces, dried for 24 h at 104° C, cooled in a desiccator, and weighed. This time was sufficient to reduce weight changes to less than 0.1% per day for the most humified samples.

9.5.2.3 *Radioactivity Measurements*

It was hoped that the age of one horizon in each core might be established by a peak in the activity of radioactive nuclides attributable to bomb tests. The most suitable nuclides, because of their relatively large amounts and long half lives, are Cs-137 and Sr-90 (Harley et al., 1965). These same reasons (which make them useful for dating) make them potentially hazardous to health, so a lot is known about their distribution in time and space (e.g. Bartlett, 1971; Cambray et al., 1971; Peirson, 1971). "Fixation" of Cs-137 in *Sphagnum* has already been reported by Bovard and Grauby (1967), though their reference to the "roots" of *Sphagnum* suggests they may have misinterpreted their results.

Slices were prepared for counting by dry ashing at 450° C with severely restricted air supply to prevent self combustion raising the temperature. It was established that loss of Cs or Sr chloride or carbonate was not detectable (less than 0.1%) in these conditions. Recovery of added Cs Cl and Sr CO_3 was nearly complete (0.99 ± 0.028; 0.97 ± 0.035; eight samples).

On most cores, absolute amounts of Cs-137 were estimated using a Ge/Li detector at liquid nitrogen temperature. These estimations were made by Mr. R.S. Cambray at the Health Physics Division of the AERE, Harwell.

On one core the spectrum of β-activity was recorded from dry ashed samples using a $^1/_4'' \times 2^1/_2''$ plastic scintillator. The energy range corresponding to a 5% window centred on the broad peak of a standard Sr-90 source of the same geometry was assumed to be mainly due to Sr-90. The counts due to natural K-40 were always less than 5% of the total in this range. On the same core, and before dry ashing the peat, relative Cs-137 amounts were estimated with a NaI detector.

9.5.2.4 *Lead Measurements*

It is known that the amount of lead in the aerial environment has increased (Jaworowski, 1967; Rühling and Tyler, 1968; Lee and Tallis, 1973; Persson, Holm, and Lidén, pers. comm.) and it seemed possible that the time of increase of lead concentration might be used for dating peat. Samples were wet-ashed with nitric acid, and lead concentration estimated by atomic absorption flame spectrophotometry. These analyses were made by Mr. M.C. French at the Monks Wood Laboratory of what was at that time the Nature Conservancy.

9.5.2.5 *Other Metals*

Interpretation of the lead data proved difficult, so additional analyses of Al, Ti, Fe, Mg, Cu, Cd, Zn, and Pb were made on a parallel core using an atomic absorption flame spectrophotometer (not available to me when the first set of Pb analyses was made). The samples were wet ashed with nitric acid. A residue of insoluble organic matter was dry ashed at 450° C and the ash combined with the nitric acid-soluble fraction. Internal standards were used, and interference in the Mg determination was suppressed with 2500 ppm $Sr(NO_3)_2$. Overall recovery (including the ashing procedure) of soluble salts of those elements analysed ranged from 0.95 to 1.02.

9.5.2.6 *Annual Growth Rates*

In one case direct estimates of growth in length and dry matter of *Sphagnum* were made using plants cut to known length and replaced in position (Clymo, 1970). In other cases innate markers (branch crowding and length) or growth against an external wire marker (Clymo, 1970) were used.

9.5.2.7 *Compression and Secondary Consolidation Parameters*

The parameter estimates were obtained from the slopes of graphs similar to those of Figure 4 and Figure 5. It should be noted that the model incorporates the functional relationships of those graphs but not the specific values of slopes.

The apparatus was a hollow rigid cylinder, 21 cm internal diameter, with perforated sides and base. In this was put a 5 cm thick slice of peat of the same diameter. A second hollow cylinder with perforated end was a loose sliding fit inside the first. The weight of the second cylinder was counterbalanced, and compressive stress applied to the peat by adding weights to the sliding cylinder. Strain was measured ± 0.01 mm against a reference mark on the central wire support of the sliding cylinder, using a travelling microscope.

The peat was saturated to field capacity before measurements began, and a polythene cover minimised evaporation.

For measurements of the short term (10 sec) time course of compression, a compressible variable resistor was put beneath the counter weight, and the output from a resistance bridge connected to the resistor was recorded. Movements up to 0.5 cm were readily recorded with trivial disturbance to the system.

9.5.2.8 *Decay Measurements*

Decay rates in the aerobic zone have been measured using *Sphagnum* in nylon mesh bags (Clymo, 1965), and for other species of peat bog plants by Heal et al. (Chap. 7). These methods are not very sensitive, and interpretation of results may be uncertain: Heal finds that the rate of loss of dry matter continues apparently unchecked whilst the respiration rate, admittedly in artificial conditions, declines. For the anaerobic zone annual loss rates may be less than 1%, and estimates from bagged material are much too inaccurate and imprecise. That anaerobic breakdown does continue is suggested, though certainly not shown, by the presence of quite large amounts of methane gas at some depth in peat, perhaps especially where bulk density is low. Any bog pool will, if stirred, yield gas bubbles, which a mass spectrometer analysis shows to contain mainly methane, nitrogen and some argon. The production of methane in waterlogged organic habitats has been known for a long time (Dalton, 1802).

Apart from methane, the only carbon-containing gas leaving the bog surface in relatively large amounts is carbon dioxide; (there may also be traces of higher paraffins or olefines). Loss of dry matter in solution may also occur in lateral runoff.

The simplest hypothesis, used here, is that carbon dioxide and solution losses come from the aerobic zone, and methane from the anaerobic zone.

The methane might, of course, have been formed at one particular stage of bog growth and simply have been trapped since, but methane is slightly soluble in water, so could have diffused upwards. The Henry's law constant for methane is 20×10^6 at 6° C (Washburn, 1926). For pressure corresponding to a depth of 5 m, gas bubbles containing 50% methane would be in equilibrium with about 1 $\mathrm{cm}^3\ \mathrm{dm}^{-3}$ in solution. Methane is given off by the bog surface. How much is oxidised by methane-oxidising bacteria is not known. Such bacteria occur in some lakes at least (Cappenberg, 1972) but Collins, D'Sylva, and Latter (Chap. 5) were unable to isolate any from Moor House peat.

Fluxes of methane and carbon dioxide were measured at twelve sites on three microhabitat types at Moor House, at roughly monthly intervals in April–October, and less often in winter, using the method described by Clymo and Reddaway (1972). This involves collecting the gas evolved from a defined area, bare of vegetation, over 24 h, and measuring the concentration of methane and carbon dioxide.

The dry matter is assumed to have the general formula of CH_2O.

Losses of dry matter in solution were estimated for the same area and time (Clymo and Reddaway, 1972).

9.5.3 Age Profiles

The age of a single horizon in each core may be obtained from the Cs-137 and β-activity profiles (Fig. 15). Both show a surface peak, a second smaller peak at about 11 to 13 cm, and declining values at greater depths. The surface peaks may perhaps be due to Cs and Sr which are effectively recycled within the live capitula and branches of *Sphagnum*, and are thus carried up with the surface. The smaller peaks are interpreted as indicating peat laid down in 1963—the peak time of fallout (Bartlett, 1971; Cambray et al., 1971). It is conceivable that the peaks relate to the Windscale nuclear reactor accident in 1957, but Windscale is about 70 km away, and Gorham (1958a) concluded that in the Lake District, which is closer to Windscale and nearly on a direct line between Windscale and Moor House, there was no clear evidence of the accident in the radionuclides accumulated in *Sphagnum*.

The apparent 2 cm difference in peak position for β-activity and absolute Cs-137 concentration is probably due to difficulty in defining the surface more accurately. The estimates were made on different cores. The cruder relative Cs-137 estimate (Fig. 15) on the core used for β-activity showed a peak at 12 cm, coinciding with that of the β-activity. It is also possible of course that the peat growth rate was slightly different in the two cores.

The concentrations are expressed per unit dry matter. If unit thickness of peat slice is used, then the lower peak is at about 16 cm. The difference is due to bulk density increasing with depth, so that a sample of given volume from lower down in the profile contains a larger amount of dry matter. Ideally the measure of radioactivity should be expressed per unit time, and the peak would then be nearer the surface than that based on unit dry matter. Assuming exponential decay of dry matter, the peak would be about 1 cm above that shown in Figure 15.

Transport of the radionuclides (other than by recycling within plants) may have occurred, just as it does during elution from an ion exchange resin column. Unfortunately there appears to be no direct experimental evidence bearing on this point. Some upward transport may

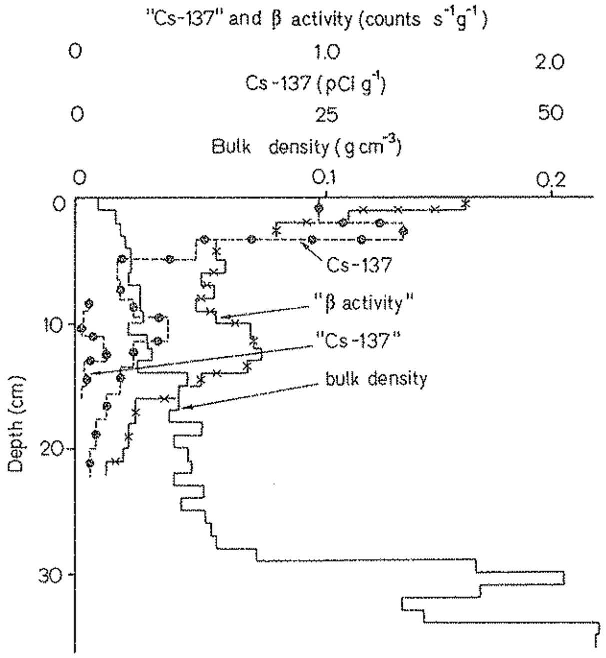


Fig. 15. Bulk density, relative "β-activity" and relative "Cs-137" activity in a core parallel to that of Figure 6. Also shown are absolute Cs-137 concentrations in a parallel core

happen on the occasions when evaporation exceeds precipitation, but in general the movement, if it occurs, is probably downwards. If the Cs-137 and Sr-90 are present as cations one would expect the monovalent Cs^+ to move more rapidly than the divalent Sr^{2+}, since the peat has a large cation exchange capacity (Clymo, 1963). If they are present as uncharged molecules this would not apply. Mattsson (1972) presents some evidence to suggest that at least part of the Cs-137 in Swedish lichen carpets is present in particulate form. The complete Cs-137 profile on a *S. magellanicum* core (Fig. 14) shows a narrow "tail" down to 50 cm, but there is no indication that the main Cs and Sr peaks in the same core are at different levels, so it is assumed that peak movement, if it has occurred, is negligible. The 12 cm level is therefore thought to have been formed in 1963.

Establishing a complete age profile involves finding some substance which has been added to the surface at a constant rate, and which has subsequently either stayed in situ or moved in a known way. Carbon-14 is not suitable for detailed continuous dating within the last hundred years, though it can be used in the same sort of way as Cs-137 for locating the bomb-test peak, as Jones and Gore (Chap. 8) show. Some of the metal elements may be suitable, but one might expect that others would not be. To sort the metals in the Moor House data into groups showing similar behaviour, the correlation coefficient matrix for the concentrations shown in Figure 6 was calculated. These coefficients were then arranged in a dendrogram using an agglomerative technique and centroid strategy (Williams et al., 1966). The dendrogram is shown in Figure 16. Al and Ti are clearly separated from the rest. Peirson et al. (1973) have evidence that Al deposited at Wraymires in the English Lake District is derived from soil, and Mattsson and Koutler-Andersson (1954) considered that both Ti and Al in a peat deposit originated from soil dust. Mg, which probably derives mainly from sea spray (Gorham, 1958b), appears similar in behaviour to Cu, Cd, and Fe. This is because Mg has a correlation of 0.65 with Fe, which in turn has a correlation of 0.28 with Cu and Cd. The correlation of Mg with Cu is only 0.01 however; Mg is linked to Cu and Cd through mutual partial similarity to Fe, but to different parts of the Fe profile.

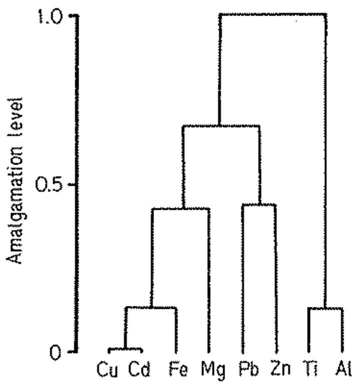


Fig. 16. Dendrogram showing relationships between the metals of Figure 6

The metals Cu, Cd, Pb, and Zn are ones whose deposition rate has increased as a result of industrial activity (Jaworowski, 1967; Goodman and Roberts, 1971; Rühling and Tyler, 1971; Lee and Tallis, 1973) even though the increase may be local and may now be declining in some cases. Peirson et al. (1973) conclude that Cu, Pb, and Zn deposited at Wraymires come mainly from artificial or industrial sources. In Table 9 the surface concentration and accumulation rate at Moor House of some of the metals are compared with those in *S. magellanicum* growing in Sweden and with total (soluble and insoluble) deposition rates at Wraymires. The Moor House figures are from the total in the top 12 cm assumed accumulated since 1963. Only Cu and Cd show notably higher concentration in the top (live) cm. The Swedish figures were calculated from 2.5 years' growth. The rates are similar, though the accumulation of Cu, Cd, and Pb is rather greater at Moor House whilst Mg and Fe are rather less; (the Zn profile is erratic for reasons not known, and may be unreliable.) The accumulation rate of Pb (63 mg

Table 9. Surface concentration (ppm) and accumulation rate (mg m^{-2} yr^{-1}) of metals on *Sphagnum* at Moor House and in S Sweden[a] and deposition rate (mg m^{-2} yr^{-1}) at Wraymires in the English Lake District[b]

Element	Al	Ti	Mg	Fe	Cu	Cd	Pb	Zn
Concentration								
Moor House	856	19	860 (740)	1,960 (2,160)	5.8[c]	6.5[c]	235	80
S Sweden	—	—	980	1,060	10.2	0.99	68[d]	90
Accumulation rate								
Moor House	292	7	215	490	8	1.1	63	42
S Sweden	—	—	281[d]	600[d]	3	0.7	45	60
Deposition rate								
Wraymires	230	—	—	280	about 30	—	55	120

[a] Rühling and Tyler (1971).
[b] Peirson et al. (1973).
[c] Concentration in the top cm about twice that in the next two cm.
[d] Calculated from other data given by Rühling and Tyler (1971).
For Moor House "surface" is the 0–3 cm layer; for S Sweden it is the surface 3 segments of *Sphagnum*.
Figures in parentheses are for total Mg and Fe in peat from *Sphagnum*-dominated unburnt blanket bog at Moor House (Gore and Allen, 1956).

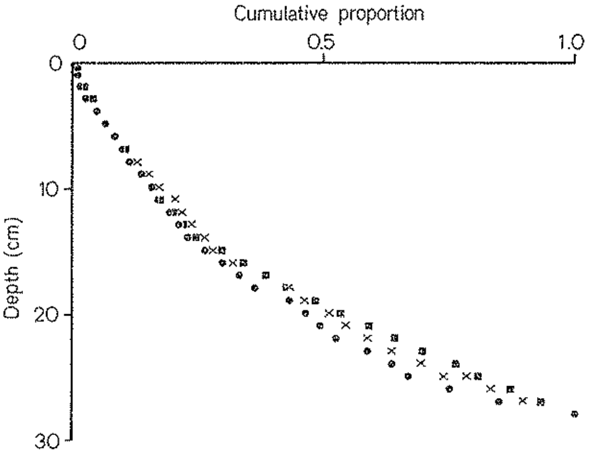


Fig. 17. Cumulative profiles for Al, Ti, and Mg, adjusted to coincide at 28 cm depth (where only the Al point is shown)

m^{-2} yr^{-1}) is similar to the deposition rate (55 mg m^{-2} yr^{-1}) at Wraymires. Chamberlain (unpubl.) calculates the average rate of emission of Pb from automobile exhausts in Britain, excluding Devon, Cornwall, W Wales and NW Scotland, to be about 20 mg m^{-2} yr^{-1}, and from smoke concentrations and particulate fallout rates in country districts he calculates Pb fallout to be on average 50 mg m^{-2} yr^{-1}. Peirson et al. (1973) conclude that Pb deposited at Wraymires is probably of both local and more distant origin. This may be the explanation for the profile of Pb concentration, which is conspicuously different from that found by Lee and Tallis (1973) at sites about 140 km away in the southern Pennines. At Moor House there are conspicuous local sources in the largely bare spoil heaps left by lead miners. The most recently worked mine, about 2 km distant, was abandoned about 70 years ago. A smelter only 0.5 km away was last used about 150 years ago (M. Rawes, pers. comm.). It must be added, however, that the climate at Moor House is not conducive to the spread of dust.

The assumption of constant rate of deposition at least to 28 cm depth seems (from Fig. 6) to be untenable for all but Al, Ti, Mg, and Pb, since the other metals show either erratic fluctuations or an increase in concentration near the surface. Although Pb shows the same general increase in concentration down to 28 cm as do Al, Ti, and Mg, the uncertainty about the relative importance of local and regional components and the likelihood that deposition rates have changed recently make it suspect.

The extent of vertical movement subsequent to deposition is uncertain. The Cs-137 and Sr-90 peak coincidence, the high cation exchange capacity, the similarity of profiles of cations of differing valence, and the abrupt changes in concentration below 28 cm, all argue against the existence of a great deal of vertical movement.

On the other hand, based on measurements of Mg concentration in rain (Clymo, unpubl.), about two thirds of the deposited Mg is not retained in the peat. Further, the total Cs-137 in the *S. magellanicum* peat profile of Figure 17 is 10.2 p Ci cm^{-2}. At Milford Haven, to the same date, 10.5 p Ci cm^{-2} were deposited (Cambray et al., 1971). Deposition is closely related to rainfall however, and whilst the mean annual rainfall at Milford Haven was 100 cm (16 yr average) that at Moor House is about 200 cm. The half life of Cs-137 is 30 yr, which accounts for only a small part (perhaps 10%) of the difference. Presumably the rest is lost in runoff. The concentration in the peat—up to about 3×10^4 p Ci g^{-1}—is unusually high, though the normalised specific activity (= activity kg^{-1} of crop/activity deposited m^{-2} d^{-1}) of 120, is not exceptional (Chamberlain, 1970).

The cumulative totals of Al, Ti, and Mg, adjusted to coincide at 28 cm, are shown in Figure 17. All three curves are similar. Assuming constant accumulation rate, limited vertical movement, and a known age for one point (12 cm), these curves may be used to give a continuous age profile.

Two features of the metal profiles (Fig. 6) remain unexplained. First is the peak in Fe at 20 cm, and second the decrease in concentration per unit dry mass of peat and for Mg, Fe, Pb, and Zn (though not for Al and Ti) in concentration per unit volume of peat at depths below 28 cm. Tyler (1972) reports a similar finding for Ca, Fe, Pb, Zn, and Cd (and less obviously for Mg and Mn). He suggests tentatively that it may be connected with oxidation reduction conditions and hydrology. This is plausible for Fe and Mn, and for Cu, Cd, Zn, and Pb which form rather insoluble sulphides but as Tyler recognises, can hardly account for a peak in Ca or Mg concentration. In the Moor House profile where a distinct change in stratigraphy is apparent, it appears preferable to postulate an earlier (below 28 cm) phase of higher productivity and/or lower decay rate. The 20 cm iron peak, uncorrelated with any stratigraphic change, may however be related to water table.

Work in progress shows that there is much variation in the profiles of metal concentration in peat; a great deal remains to be discovered about the factors controlling vertical movement of metals in peat-forming systems.

Note added in proof. Four years later it is apparent that the simple concept of static elements and concentration increase as a result of decay of the organic matrix is wholly wrong for many elements in the anaerobic zone, although it may not be too incorrect for the depths to which it has been used here, and especially for Mg.